Atoms and Photons and Quanta, Oh My!

Ask the physicist about atomic, nuclear, and quantum physics

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Morgan & Claypool Publishers

ISBN 978-1-6270-5939-8 (ebook)
ISBN 978-1-6270-5938-1 (print)
ISBN 978-1-6270-5941-1 (mobi)

DOI 10.1088/978-1-6270-5939-8

Version: 20150601

IOP Concise Physics
ISSN 2053-2571 (online)
ISSN 2054-7307 (print)

A Morgan & Claypool publication as part of IOP Concise Physics
Published by Morgan & Claypool Publishers, 40 Oak Drive, San Rafael, CA, 94903, USA

IOP Publishing, Temple Circus, Temple Way, Bristol BS1 6HG, UK

For Geoff, Andy, Hannah and Adam.
You all have made me proud to be your dad.

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Preface

I am *The Physicist*! Since 2006 I have run a web site, www.AskThePhysicist.com, where I answer questions about physics. The site is not intended for answering highly technical questions; rather the purpose is to answer, with as little mathematics and formalism as possible, questions from intelligent and curious lay persons. For several years before my retirement from the University of Georgia I ran a similar Q&A site for the Department of Physics and Astronomy there. Over the last decade I have answered more than 5000 questions on line and uncounted more by brief email replies. I have found this very rewarding because it is an extension of my more than 40 years experience teaching and because I learn something new almost every day. The questions I receive reveal what aspects of physics interest people and what principles they do not grasp. They reveal a wide-spread thirst to understand how physics describes, on many levels, how our Universe works. It is gratifying that the site has on the order of 50–100 000 visits per month, far more than the number of questions asked; I interpret this to mean that there are many visits by people who simply like to read and learn.

This is the second book in the *Ask the Physicist* series. The first book, *From Newton to Einstein, Ask the Physicist about Mechanics and Relativity*, would provide an excellent foundation for this book; that is, it would be better if you read the two books in sequence. While it is certainly possible to get a lot out of this book without having read the first, be warned that some basic understanding of classical mechanics and the theories of special and general relativity are assumed for many concepts in this book.

Although this is a book about topics in 'modern physics', it is impossible to appreciate both the roots of quantum physics and its applications without having an understanding of the nature of light, or more generally electromagnetic radiation. And to understand the nature of light, an understanding of electricity and magnetism is needed. To that end, chapter 1 of this book will introduce the electromagnetic concepts required to understand the basics of light needed to understand the foundations of quantum physics.

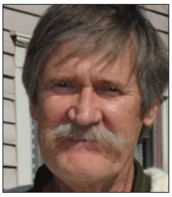
The bulk of the book is devoted to chapters which will contain mainly categorized groups of Q&As from the web site, sort of a *Best of Ask the Physicist*. Enjoy and learn!

Acknowledgements

With the internet at my fingertips and a few classic texts I have held on to, I can usually find the answer to any reasonable layman's question in a few minutes. I am particularly indebted to the ROMEOs (retired old men eating out) with whom I frequently discuss interesting *Ask the Physicist* questions at our Wednesday lunches. I should also acknowledge the visitors to the site who have submitted scores of questions which have challenged me to seriously research the topic or brush up on some physics I have not thought about for years; I learn something new or relearn something old almost every day.

Author biography

F Todd Baker



'The Physicist' is F Todd Baker. He received AB and MA degrees from Miami University and a PhD degree from the University of Michigan. His area of research is nuclear physics and he has published more than 70 articles in refereed journals as well as made numerous presentations at conferences and workshops. He has more than 35 years of college and university teaching experience. In 2006 he retired from the University of Georgia where he taught and performed nuclear physics research for 32 years. Previously he held a postdoctoral research associate position at Rutgers University and teaching positions at Carroll College (Wisconsin) and St Lawrence University. He now lives in Athens, Georgia with his wife Sara in a 100-year-old house mainly restored by him and decorated and landscaped by her. He has four beloved children aged 18–45 years. He enjoys bicycling around town, playing violin, cooking and baking, outdoor activities, DIY projects, film, music of many genres, working puzzles, reading mainly European murder mysteries and hanging out in coffee houses. His *Curriculum Vitae* may be seen at www.ftoddbaker.com/cv.html.

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Chapter 1

Let there be light!

1.1 Introduction

The main emphasis of this book is to provide an accessible introduction to quantum physics, atomic physics and nuclear physics. But, to appreciate the roots of quantum physics an understanding of 'light' is imperative. Although light is colloquially used to denote that to which our eyes are sensitive, it is now recognized that visible light is only a tiny fraction of a continuous spectrum also containing 'invisible light'. The entire spectrum is referred to as electromagnetic radiation. Often when physicists talk about light they are really talking about the whole spectrum, for example references to the *speed of light*. The task, then, before proceeding with the main topics of this book, is to provide a brief exposition of the history of our understanding of the nature of light from the end of the 19th century. To accomplish this we will also have to acquire a basic understanding of electricity and magnetism.

1.2 History

Man has wondered for millennia what the nature of light is. Prior to the 17th century, before experimental methods were used, light was another aspect of philosophy. Most models, in one way or another, assumed light to be some sort of particle. Isaac Newton (1642–1726), in addition to his origination of Newtonian mechanics, studied the nature of light. He was the first to realize that white light was, in fact, a mixture of all the colors. This he demonstrated by using a prism to split sunlight into a rainbow. In 1704 Newton published his theory of light, *Opticks*, in which he postulated light to be composed of particles. This model choice was influenced by earlier particle theories, notably that of the French philosopher Pierre Gassendi (1592–1655), whose work Newton had studied in his youth. The particles corresponding to different colors Newton imagined to have different masses. Two aspects of particle theories are difficult to reconcile with the known properties of light. One problem is that, to understand refraction (the bending of light rays when entering a denser medium)

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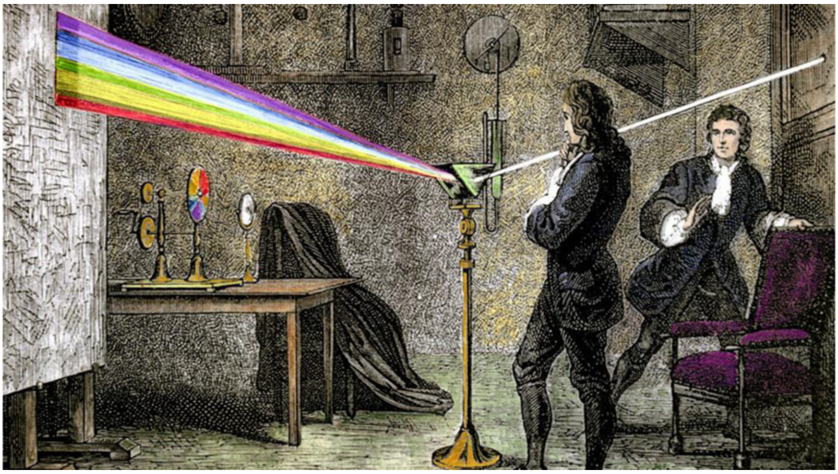

required that the light travel faster in the denser medium. Since, in Newton's time, technology did not exist to accurately measure the speed of light anywhere, this could have been correct; in retrospect, we know the reverse is true. The second difficulty is in explaining the phenomenon of diffraction. If the shadow of a sharp straight edge is closely examined, it is seen that the shadow is not discontinuously sharp but slightly blurred. This is similar to the way water waves will pass through an aperture but spread after passing through—diffraction. Newton was able to argue that when the particles passed close to the edges they caused disturbances in the æther causing the particles, which normally traveled in straight lines, to be deflected. (The æther was the medium thought, from the time of Classical Greece, to permeate all space above the Earth.) Let us use a Q&A from AskThePhysicist.com to introduce an alternative view of what light might be—a wave.

> **Question:** Who first measured the wavelength of light and how?
> **Answer:** Credit is usually given to Dr Thomas Young (1773–1829). Most famous of his 'measurements' is the so-called double slit diffraction experiment in which light strikes two closely spaced slits in an opaque sheet. Each slit acts like a source of waves and these two waves interfere with each other when they strike a screen some distance away. What interference means, essentially, is that the two waves add up to give a net disturbance at that point. So, if the two waves are both at a crest they add up to look twice as big (therefore bright) but if one is at a crest and the other is at a trough, they will cancel each other out and there will be no light. Young explained many well-known optical phenomena (such as the colored fringes you see from an oil slick) using the idea of interference; he and his ideas were, as often happens with revolutionaries, reviled by many of his contemporaries. Some 13 years after Young had described interference, Augustin Jean Fresnel (1788–1827) independently developed the notion of interference.

Although other prominent thinkers, e.g. Robert Hooke (1635–1703) and Christiaan Huygens (1629–95), proposed wave models to describe light, Newton's reputation ensured that the particle theory would be most accepted during the 18th century. In 1799 when Thomas Young first presented his idea to the Royal Society that light was comprised of waves, not particles, he was ridiculed because Newton was so revered. However, he persevered and in 1803 he presented the results of his experiments which pretty much nailed it for the wave picture of light. The crucial experiment was the double-slit experiment as noted above. Because this experiment is crucial to the history of light and will be just as crucial to understanding the development of quantum physics, it warrants some detailed discussion here.

The crucial idea is that if there are two waves traveling in the same medium and they meet at some point, the disturbance at that point is the sum of the two waves. So, if one is at a crest and the other is at a trough, there will be no net wave there! This is called destructive interference and the two waves are said to be *out of phase* with each other. For this to work, the waves must be identical in shape, speed, wavelength and frequency. Wavelength is the distance between two adjacent crests and frequency is the rate at which crests pass you, crests per second; there is a simple relationship among speed (v), wavelength (λ) and frequency (f), $v = \lambda f$. We usually think of the waves as having a simple undulating shape ∿∿ (sinusoidal, for readers familiar with trigonometry), but any shape is ok if it is periodic (repeats itself so we can identify a wavelength) and the shapes of the two waves are identical. Identical waves are called *coherent*. Now, it is a little tricky to make two coherent sources so that you can study their interference. But, there is a clever way to achieve this. Imagine water waves moving across the surface of a lake and you put up a wall to stop the waves. Now make two small openings in the wall and each will behave like a source coherent with the other. An example for water waves is shown in figure 1.1. Note that there are directions where the propagating waves are very small (the dashed line shows one) and where they are very large (solid line). If this were light and there was a screen upon which the light was shining, you would see alternating dark and bright bands.

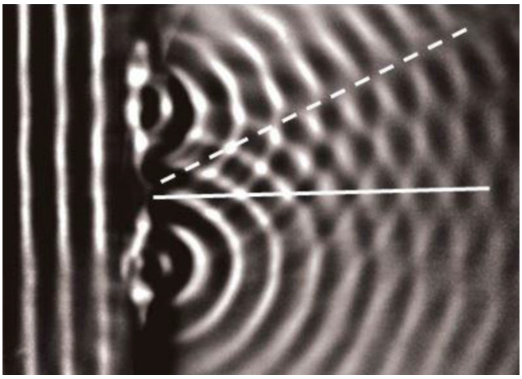

Figure 1.1. Double-slit water wave diffraction.

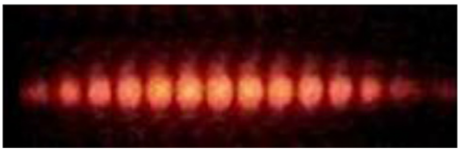

Figure 1.2. Double-slit diffraction pattern for red light.

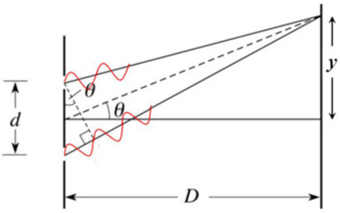

Figure 1.3. Double-slit geometry.

This is what Young did. Illuminating the slits with a single wavelength, the pattern shown in figure 1.2 results. So, if we could understand how the spacing between the spots is related to the wavelength, we could use this experiment to determine the wavelength of the light.

To understand how the spacing of dark and bright spots can be related to the wavelength of the waves, have a look at figure 1.3. Focus your attention on one spot on the screen, on the right of the figure; you can specify where this spot is by specifying either the distance y or the angle θ. The two slits, on the left, are separated from each other by a distance d and from the screen by a distance D. Your point on the screen is then illuminated by the rays, drawn in the figure, from the slits. These two rays are of different lengths (unless $y = 0$) and the difference in path lengths is $d \sin\theta$. Now, suppose that this path difference is exactly one full wavelength as I have drawn. The two waves will arrive perfectly in phase and there will be a bright spot there. The same would be true if I chose an angle where exactly two, or three, or four, etc, wavelengths would fit; in every case the resulting spot would be bright. So, the condition for constructive interference is simply $n\lambda = d \sin\theta$ where λ is the wavelength and n is any integer, 0, ±1, ±2, ±3, … The ± just means that this will happen both above and below the center of the screen. (It is not important to understand the details, but some approximations are made which depend on the distance to the screen being very large compared to the spacing of the slits. In the real world, this is almost always true. It also allows for a simplification that $\sin\theta$ is approximately equal to y/D.) Now you see why Young is given credit for first measuring the wavelength of light—just measuring the positions of the bright spots allows you to deduce the wavelength. For example, for the fourth bright spot from the center you can write $\lambda = yd/(4D)$. It turns out that wavelengths of visible light are very small; the visible spectrum spans approximately from 400 nm (violet) to 700 nm (red), a nm (nanometer) being one billionth (10^{-9}) of a meter.

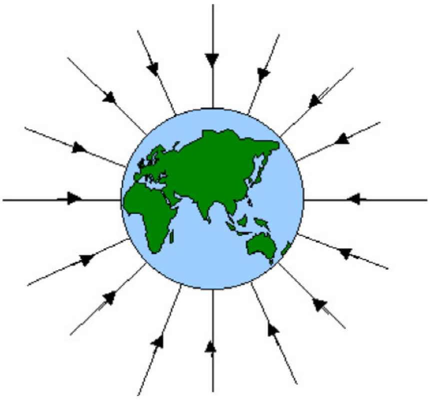

Figure 1.4. Earth's gravitational field.

1.3 What are force fields?

Now that you understand that light has been demonstrated to be waves, the obvious question is 'what is waving?' Alas, here is another detour which we are required to take since I must assume that you, the reader, are not really conversant with electric and magnetic fields, maybe not even the concept of a field. (A teaser: electric and magnetic fields are what are waving!)

The easiest force field to visualize is the Earth's gravitational field because we are all familiar with that. A large mass like the Earth will exert a force on some smaller mass which points straight toward the center of the Earth and becomes weaker as we move farther from the Earth; we know that the force is inversely proportional to the square of the distance from the center of the Earth. Figure 1.4 shows how we draw a picture which represents this idea and the same general idea applies to other fields: the lines have arrows attached with the arrows showing the direction of the force felt in this field and the density of the lines tell you how big the force is (note that the lines become farther apart as you move out). The idea of a field was first conceived by Michael Faraday (1791–1867), a brilliant investigator of electromagnetism and, interestingly, almost entirely self-educated. It may seem that the field is just a clever picture to depict what would happen if something sensitive to the field (a mass in the case of gravity) is inserted. A better way to think about fields, though, is that the field actually represents a modification of the space around the source of the field. The field is 'real' and a field actually contains energy.

1.4 Electric and magnetic fields

Now, what we are really interested in here are electromagnetic fields. Just as mass is the source of gravitational fields, electric charge is the source of electric fields. And, electric fields behave like gravitational fields in that if you have a sphere of electric charge its field is radial (points either toward or away from the center of the sphere) and is inversely proportional to the square of the distance from the

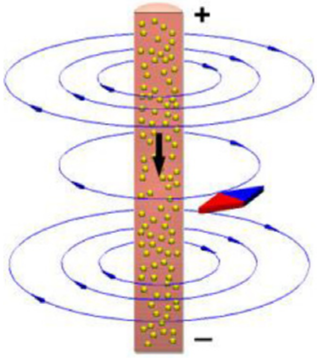

Figure 1.5. Magnetic field of a current.

center of the sphere. But perhaps the biggest difference between electric and gravitational forces is that there is only one kind of mass but there are two different kinds of electric charge. We find empirically that like kinds of charge repel each other (which never happens in gravity) and different kinds attract. We arbitrarily assign a sign of either plus or minus to the two kinds of charge to distinguish them and say that the field of a positive charge points radially outward; the field of negative charges therefore points radially inward. This means that when you see an electric field it tells you the direction in which a positive electric charge would move in this field; a negative charge would move opposite the field direction. So, the field lines in figure 1.5 could represent electric field lines if the Earth were negatively charged (it actually is). The assignment of the sign of electric charge was first made (for no known good reason) by Benjamin Franklin (1705–90) such that the electron charge is negative and the proton charge is positive and of equal magnitude $e = 1.6 \times 10^{-19}$ coulombs (C). See appendix A for the formal definition of the coulomb.

In addition to the electric field, another seemingly different field appears if electric charges are moving; this is the magnetic field. So, if you cause many electric charges to flow through a wire, as we do whenever we turn on an electric light, the moving charges constitute what we call an electric current and electric currents are the source of magnetic fields. Figure 1.5 shows the magnetic field caused by a current-carrying wire. Since you are probably familiar with magnets and compasses which do not seem at all like they are caused by electric currents, figure 1.5 shows how a small compass or bar magnet would behave in this field: it aligns with the field so that the north pole of the magnet (shown in red) points in the direction of the field. You could think of the north pole of the magnet as a positive 'magnetic charge' and the south pole as negative to see how this is really a different kind of field from the electric field; if you just put an electric charge at rest somewhere in the magnetic field, it would feel no force at all.

Although it does not concern us here, it should nevertheless be noted that electric and magnetic fields are really both just different aspects of a more overreaching concept, the electromagnetic field.

1.5 Maxwell's equations

During the 18th and 19th centuries many scientists studied the properties of electric and magnetic forces as well as the relations between the two. Around 1870 James Clerk Maxwell (1831–79) took all that was known about electromagnetism and condensed it into his four famous equations. A summary, in words, of the essence of Maxwell's equations can be found in one of my *Ask the Physicist* answers:

> **Answer:** The laws of electromagnetism are perfectly symmetric: a changing magnetic field causes an electric field and a changing electric field causes a magnetic field. The first of these is called Faraday's law and the second is part of Ampere's law. You seem to think that only a permanent magnet is magnetism. In fact, any moving electric charge causes a magnetic field. The most common source of magnetic fields is simply an electric current. Here are some facts about electric and magnetic fields:
>
> - electric charges cause electric fields,
> - electric currents (moving charges) cause magnetic fields,
> - changing magnetic fields cause electric fields, and
> - changing electric fields cause magnetic fields.

An amazing result derived from Maxwell's equations was that they predicted the existence of waves of electric and magnetic fields. (You may see a concise derivation at http://askthephysicist.com/thespeedoflight.pdf.) The speed c of these waves was found to be determined by only two well-known constants (see appendix A), the permittivity of free space ε_0 and the permeability of free space μ_0, $c = 1/\sqrt{(\varepsilon_0\mu_0)} = 3 \times 10^8\ \mathrm{m\,s^{-1}}$. Essentially, these constants quantify the strengths of electric and magnetic fields.

So now, finally, we know what is waving—electric and magnetic fields. Figure 1.6 shows a simple electromagnetic wave with its electric and magnetic fields. Realizing that this shape is moving to the right, you can see that both fields will be changing at any point of space and, in accordance with Maxwell's equations, may be thought of as 'causing each other' and maintaining the flow of energy along the direction of propagation.

The entire electromagnetic spectrum is shown in figure 1.7, demonstrating that only a tiny fraction of it is visible. To create electromagnetic waves, all that is needed

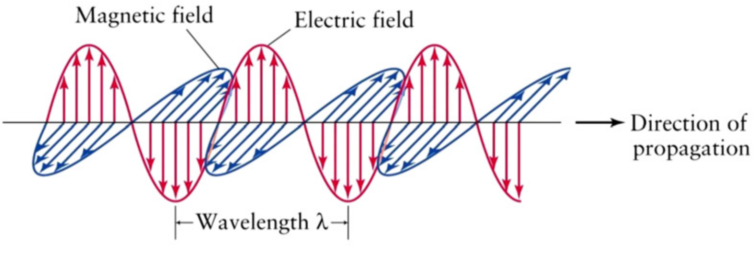

Figure 1.6. Electromagnetic wave.

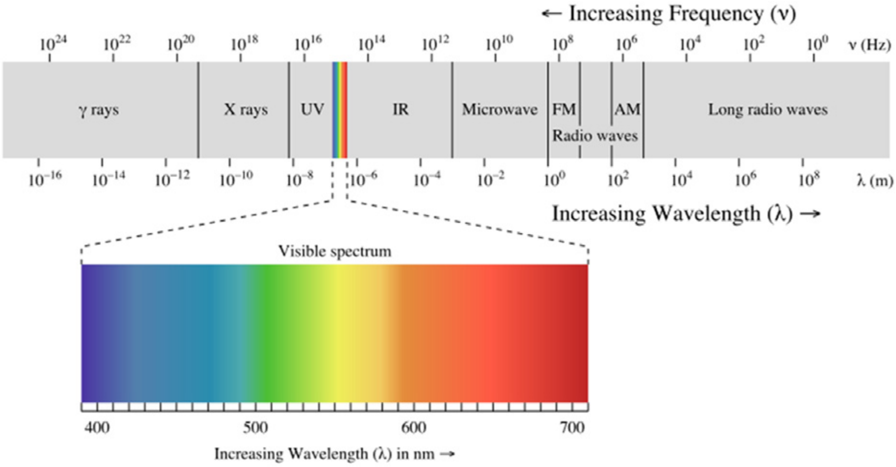

Figure 1.7. The spectrum of electromagnetic waves.

is an accelerated electric charge. The way a radio antenna works is to simply have electric currents running back and forth in a metal rod; in order to change the direction of the currents, the electrons must accelerate and the result is radio waves which propagate away from the rod. Similarly, an atom or molecule can act as an antenna because it is made up of electric charges which can be induced to oscillate.

Finally, it should be noted that electromagnetic waves carry energy. This should not be surprising since we feel warmth when sunlight shines on us. The Sun is the source of virtually all energy on Earth. Fossil fuels are merely long-stored solar energy. The energy of an electromagnetic wave is proportional to the square of its electric field. So a wave with twice the electric field of another carries four times the energy.

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Chapter 2

Quanta and photons, oh my!

2.1 Prologue

Question: Some time ago I found a quantum lecture series on the web. I thought that even if I didn't 'get' all the math, I would still perhaps learn something. Well, the lectures were 'here is some math you will need for quantum mechanics', 'here is some more math you will need, etc'. I skipped ahead many lectures and found the next lecture was 'here is some MORE math you need, etc'. I am wondering this: is quantum mechanics something like the Ptolemaic system where the math works even though there is no REAL understanding of the underlying phenomena?
Answer: No question—quantum mechanics is quite mathematical as are most advanced topics in physics. It's the nature of the beast, I guess. But it should be possible to convey the essence of just about any topic in physics with minimal mathematics or even none. That, after all, is what I aim for at AskThePhysicist.com. So, is quantum mechanics comparable as a theory to the Ptolemaic description of the solar system? I would say certainly not. Originally Ptolemy postulated circular orbits for the planets and satellites which did a fairly adequate job for the 2nd century. But then there were more observations which were not described and one had to add epicycles to the Ptolemaic system, sort of circles on circles. It all became contrived to just fit the data and eventually was given up as a failed idea. Quantum mechanics, on the other hand, matured rather quickly from its sketchy origins (black body radiation, photoelectric effect, Bohr model, Compton scattering) to what today is called nonrelativistic quantum mechanics. If applied to experimental results over a time period of about 80 years, the theory has never failed to be correct. But just because it works wonderfully well does not mean we really understand it. I think most physicists would agree that we do not have an answer as to 'why it works' or whether there is an underlying possibly more deterministic physics. Certainly Einstein was never comfortable with quantum mechanics. To compare it to the purely empirical Ptolemaic 'theory' really sells it short.

doi:10.1088/978-1-6270-5939-8ch2

2.2 Introduction

Physicists were pretty smug at the end of the 19th century. The great successes in understanding electromagnetism, classical mechanics and thermodynamics had induced a state of complacency. There was a general feeling that very little was left to be understood and that the main task of physicists was to be making more accurate measurements of quantities already well understood. But exciting frontiers were just about to open and a century of amazing progress in understanding the world would result. Classical electromagnetism, briefly summarized in chapter 1, was to be at the heart of much of this totally new physics. The first of the *Ask the Physicist* books, *From Newton to Einstein*, addressed the theories of relativity. In this book we will learn about the development of quantum physics and the topics it naturally spawned—atomic physics, nuclear physics and particle physics. The pivot point which launched quantum physics was the failure of classical physics to describe the glowing of hot objects.

If you are not familiar with the electron-volt (eV) as a unit of energy, you should read appendix D before proceeding. And, if you need a refresher on energy itself, see appendix B.

2.3 Black-body radiation

Recall from chapter 1 that Newton discovered that light from the Sun was a continuous spectrum of light, a rainbow of colors, which was later understood to be composed of waves of all wavelengths. But they are not all of equal brightness. Indeed, if careful measurements are made, it is found that all wavelengths present are distributed as shown by the curve labeled 5000 K in figure 2.1. If you are not

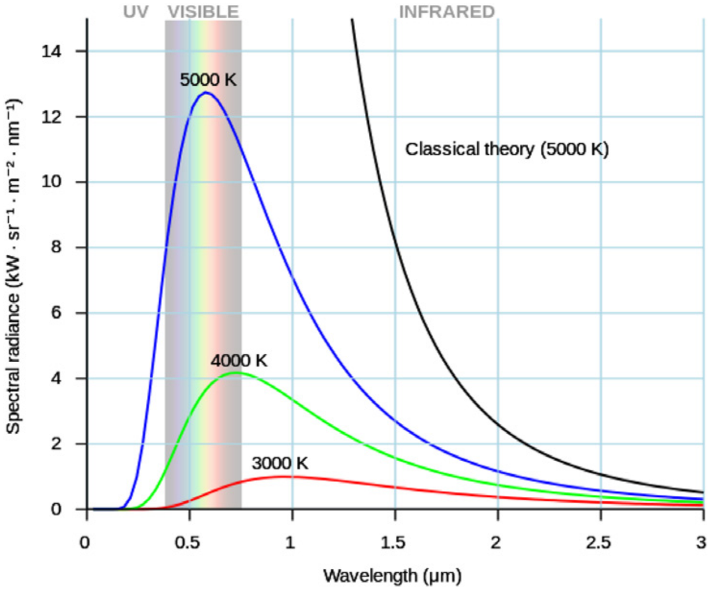

Figure 2.1. Black-body spectra for various temperatures.

familiar with the Kelvin temperature scale, room temperature is about 293 K, so 5000 K is very hot. What is plotted is the intensity or brightness of glowing objects, specifically objects called black bodies. Note that the Sun, a pretty good approximation of a black body, is most intense around the color yellow meaning that its surface temperature is around 5000 K. The reason that we evolved to have eyes most sensitive to wavelengths near yellow is because, since the Sun has always been our primary source of light, most light we see has wavelengths near yellow. If we had evolved near a star whose temperature was closer to 4000 K, our eyes would probably have evolved to be most sensitive to infrared radiation.

> **Question:** I have a question about black-body radiation. What exactly is a black body, and how can it theoretically emit infinite energy, and how is this related to black-body radiation?
> **Answer:** A black body is a perfect absorber of radiation. Of course, there is no such thing, but many things are excellent approximations. One example which is very nearly a black body is a tiny hole in a cavity: radiation which enters the hole is highly unlikely to find its way back out the hole. If you have a black body, it will also radiate energy which is called black-body radiation. So, if you want to study black-body radiation, look at the light coming out of the tiny hole in a hollow metal ball. It turns out that the spectrum has a continuous distribution of energies and is brightest at a wavelength which becomes smaller as the object becomes hotter; for example, red radiation is not 'as hot' as blue radiation. Black-body radiation played a very important role in physics. At the end of the 19th century the nature of black-body radiation was well known and physicists were trying to explain it. It turns out that classical physics predicted a spectrum which contained an infinite amount of energy, obviously not possible (that must be what you are referring to). This was referred to as the 'ultraviolet catastrophe' because the spectrum just became bigger without bound for small wavelengths. The problem was finally solved by Max Planck by proposing the first ever application of quantum physics.

This Q&A tells you what a black body is and, essentially, as is often the case in physics, it is an idealization, but one for which calculations can be conveniently performed. Stars turn out to be pretty good black bodies which is how we determine their temperatures. Shown in figure 2.1 are the shapes of measured black-body spectra at temperatures T of 5000 K, 4000 K and 3000 K. As you can see, the hotter the black body, the more energy is radiated but as it cools the radiation becomes redder. In fact, it was well known empirically that the power output of a black body is proportional to T^4—twice the temperature gives 16 times more energy; this is the Stefan–Boltzmann law. Such nice data were a natural inspiration for the physics of the day to try to explain them. Although it is well beyond the scope of this book to describe the analysis based on Maxwell's equations and statistical thermodynamics, it was a fairly straightforward task to calculate the energy spectrum for a black body and the result was called the Rayleigh–Jeans law; essentially the result was that the power P (rate of energy radiation) radiated by a black body at some temperature and at a particular wavelength λ was inversely proportional to the fourth power of the wavelength, $P \propto 1/\lambda^4$. Of course this means that the smaller λ is, the larger P is so that at very short wavelengths the energy being radiated became infinite. The total power of radiation for any black body was predicted to be infinite by the physics of the day. There were no errors in this analysis, the best minds of the day all agreed that the analysis was correct. This could only mean that the physics was incorrect. Since short wavelengths are called ultraviolet, this failure was dubbed the *ultraviolet catastrophe*. Figure 2.1 shows the predicted behavior of the radiation curve for 5000 K, labeled 'classical theory'.

The model behind these calculations imagined that the electrons in the black body were oscillating and behaving like little antennas with all possible frequencies and an oscillating electric charge radiates electromagnetic waves and, when in equilibrium with its environment, also absorbs other electromagnetic waves and the population of oscillators at each frequency could be predicted by statistical considerations which were well understood from thermodynamics. Now, Max Planck (1858–1947), studying this problem, tried the following guess. Suppose that any oscillating charge in the black body could not oscillate with just any old energy; instead, an oscillator with frequency f could only have an energy which was $E_n = hf$, $2hf$, $3hf$, $4hf\ldots$ where h is some constant. Now, using h as an adjustable parameter, he found that he was able to perfectly describe the data at any temperature. He found that $h = 6.6 \times 10^{-34}\ \mathrm{J \cdot s} = 4.1 \times 10^{-15}\ \mathrm{eV \cdot s}$ in order to fit the data. This was a revolutionary discovery; it was akin to saying that a 1 kg mass hanging on a spring could oscillate with an amplitude of 1 cm or 2 cm but that it would be physically impossible to have any amplitude between. For example, the frequency of yellow light of about 600 nm would have a frequency of about $f = 3 \times 10^8/6 \times 10^{-7} = 5 \times 10^{14}$ Hz; so the step between two energy states of the oscillator would be about $hf = 6.6 \times 10^{-34} \times 5 \times 10^{14} \approx 3 \times 10^{-19}\ \mathrm{J} \approx 2\ \mathrm{eV}$. And we can now envision how such an oscillator could radiate energy, by dropping from one energy state (nhf) to the next lowest state $[(n-1)hf]$; the radiation itself could be viewed as little bundles of energy hf but any particular oscillator would always radiate at the same frequency. Here is a startling thing—light is composed of particles, not waves? Newton was right after all? We will have more to say about this later in this chapter, but don't count waves out yet. So, nobody took this notion of light particles seriously, not even Planck himself who said 'The theory of light would be thrown back not by decades, but by centuries, into the age when Christian Huygens dared to fight against the mighty emission theory of Isaac Newton…'. Quite simply, classical electromagnetism said that an oscillating electric charge would radiate away its energy with that frequency until all its energy was gone unless it absorbed some energy from the environment.

Planck's hypothesis for solving the black-body problem represents the birth of what is today called quantum mechanics. The oscillators are said to be quantized and the particles of light are called photons. Here is the first indication that at very small sizes, comparable to the sizes of atoms, Newtonian mechanics and Maxwellian electrodynamics are wrong or at least incomplete. Recall from the first book of the *Ask the Physicist* series, *From Newton to Einstein*, that a similar situation was found for objects traveling at very high speeds, comparable to the speed of light, requiring the development of the special theory of relativity.

2.4 The photoelectric effect

Imagine shining light on a piece of metal. There are electrons in the metal, of course. Light carries energy and it takes energy to extract electrons from a metal, so we would expect that light would cause electrons to be ejected from the metal. Imagine the electrons experiencing the wave; they respond to the electric field by starting to move so with each passing wave front they move faster and faster and eventually have acquired enough kinetic energy to pop out of the metal. So, if you wait long enough, electrons should start popping out. Or, if you do not want to wait too long, just increase the intensity of the wave (brighter) which will cause the electron to experience larger fields to accelerate it up to speed. This ejection of electrons is called the photoelectric effect and it was indeed observed in the late 19th century. However, the behavior was not at all as expected: only very short wavelengths would cause electrons to be ejected, e.g. ultraviolet light would do the trick. But if you shone red light on the metal, no matter how intense or how long, electrons were never ejected. And, if the wavelength was capable of causing electron emission, it happened instantaneously no matter how low the intensity. This was a puzzle and, like black-body radiation, seemed to indicate a problem with electromagnetic theory.

Figure 2.2 shows a schematic sketch of an experiment to observe the photoelectric effect. A glass tube contains two metal plates and is evacuated. Wires connected to each plate complete a circuit. Light strikes the emitter plate and electrons are ejected. Since the ejected electrons have kinetic energy, they move across the gap to the collector plate. The battery is not required at all; just by connecting the two plates by the wire a current would flow and be observable by the ammeter. The purpose of the battery is to determine the energies of the ejected electrons. Note that the battery is connected such that the collector plate is negative and the emitter plate is positive; this results in the electrons being repelled by the collector plate. So, as the voltage of

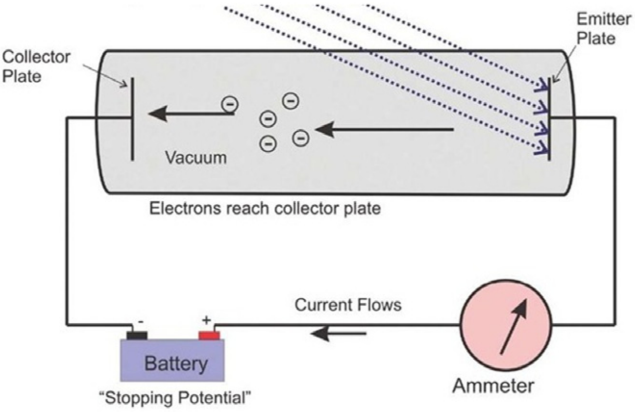

Figure 2.2. The photoelectric experiment.

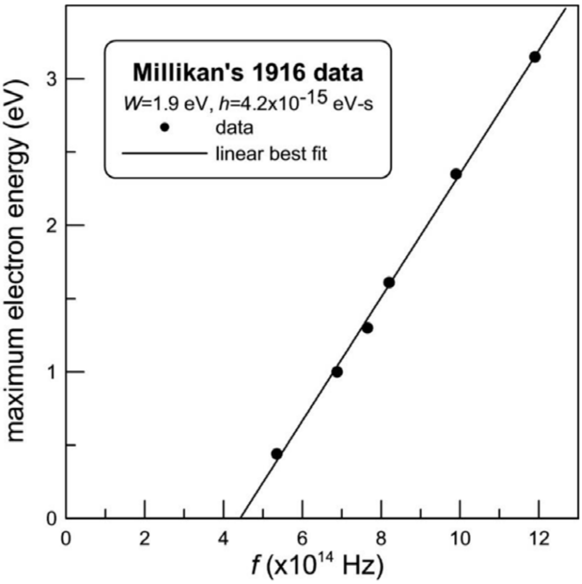

Figure 2.3. Photoelectric effect data for sodium.

the battery is gradually increased, the electrons reach the collector plate with smaller and smaller speeds until eventually, they do not reach at all and the current ceases. This then allows the measurement of the energy of the ejected electrons. For example, if with a particular light frequency and intensity the current ceases when the battery voltage is 2.4 V, the maximum kinetic energy of electrons being ejected by that light is 2.4 eV. Furthermore, increasing the intensity, had no effect on this maximum kinetic energy, only on the amount of current which flowed.

Question: I am trying to teach quantum physics for the first time. I am not sure what the link is between the photoelectric effect and Planck's constant or how to link it meaningfully for the students.
Answer: The graph in figure 2.3 shows photoelectric effect data. Plotting the maximum kinetic energy of the electrons as a function of the frequency f of the light which caused them to be ejected yields a straight line. But, to understand why the data behave like they do, you must assume that the energy given to the electrons is from photons, not try to understand it in terms of waves. Now, the photon gives all its energy to the electron and, since it takes a certain amount of energy, call it W, to remove an electron from the metal, then the kinetic energy K of the electrons must be $K = hf - W$ where h is the slope of the line. It just so happens that h (the slope of the line) turns out to be Planck's constant, and that is the link you seek.

In 1905 Albert Einstein proposed an explanation of the photoelectric effect. Following up on Planck's work, Einstein suggested that the radiation was indeed composed of massless particles called photons and each photon had an energy

$E = hf$. In the photoelectric effect, each photon either interacted by giving all its energy to a single electron or else not at all. So, each electron acquires a kinetic energy of hf. But it takes energy to remove an electron from a metal, otherwise electrons would simply float away from the material in which they reside. That energy is called the work function of the metal (a pretty strange name!) which I will denote as W. Typically, W is on the order of 2–5 eV. Therefore the ejected electron has a kinetic energy of $K = hf - W$; note that this hypothesis finds that K is a linear function of f, so if measurements of K are made for several wavelengths and K is plotted as a function of f, a straight line should result. If Einstein's idea is correct, it is evident why the photoelectric effect does not happen for red light: red photons have less than 2 eV of energy, so although they may give that energy to electrons, the electrons do not have enough energy to overcome the work function. The data collected in 1916 by the American physicist Robert Millikan (1868–1953) are shown in figure 2.3; the equation of the best straight line through the data points is $K = 4.2 \times 10^{-15} f - 1.9$. This is just about the same as the value which Planck deduced from his analysis of black-body radiation and the separately measured work function for sodium is about 2.2 eV. Einstein and Millikan received the Nobel Prize in 1923 for this work.

So, it now begins to look like there really is something to this quantum business. This is just the beginning of more than 40 years of developing quantum mechanics as a formal theory, learning about the microscopic world—atoms and nuclei. The next section describes yet another early experiment indicating that light, in its interaction with matter, behaves like particles.

2.5 Compton scattering

Question: I am a high school physics teacher and am having some difficulty with part of the Compton effect. I am trying to come to grips with whether the collision between the x-ray and the electron is elastic or inelastic? From what I have been able to find on the subject, when the x-ray collides with the 'whole' atom it results in an elastic collision and the x-ray leaves with the same frequency with which it came in with. On the other hand, when the x-ray collides with something closer to its own mass (an electron), it results in an inelastic collision and the x-ray is ejected with a lower frequency and energy. Any help you could provide would be greatly appreciated. I hate to think that I am not teaching it correctly and sending my students out into the world with misconceptions imparted to them by me.

Answer: The Compton effect is elastic scattering of photons from some mass. Elastic does not mean that the energy of the incoming particle remains constant, it means that the sum of the energies of the incoming particle and the target remain constant. Assuming the target is at rest before the collision, after the collision it will recoil and carry away some of the energy which the photon brought in and the only place it can obtain this energy is from the photon. It is maybe easier to see this by thinking about classical particles. If a BB (photon) hits a bowling ball (whole atom), the bowling ball is almost at rest after being hit and therefore the BB has approximately the same energy (and speed) after the collision. If the BB

> (photon) hits a marble (electron), the marble will be moving after the collision so the BB must have lost energy (and speed). Both processes are elastic. An inelastic collision is one in which the total energy before and after are not equal.

Photons, if they are indeed particles, should scatter from other particles. So, suppose that we have an electron and we ‘shine light’ on it; in other words, let us bounce the photons off of electrons and look at the scattered photons. What should happen is called an elastic collision, a collision in which both the total energy and the total linear momentum are conserved. Such an experiment was performed by American physicist Arthur Holly Compton (1892–1962) in 1923; he was awarded the Nobel Prize in physics in 1927 in recognition of this work.

Imagine a head-on collision between two balls of unequal mass, the heavier one at rest before the collision. If the collision is elastic, the heavier ball will be moving in the direction in which the lighter ball came in and the lighter ball will have bounced back in the opposite direction but with a speed slower than the speed it originally had. Therefore, if we were able to watch only the lighter ball, we could tell that it had collided with something because it would have different energy and momentum after the collision. For example, if the incident ball had a speed of v before the collision and had half the mass of the other ball, it would rebound backwards with speed $v/3$ and the ball originally at rest would move forward with speed $2v/3$. These calculations assume that momentum is $p = mv$ and energy is $E = \frac{1}{2}mv^2$. The details are presented in appendix C.

How a photon would recoil from an electron is different from the two balls in the example above. This is because, since a photon has no mass but still has energy and momentum, momentum (mv) and energy ($\frac{1}{2}mv^2$) must be redefined using special relativity. Similarly, the energy is no longer $\frac{1}{2}mv^2$. As was shown in *From Newton to Einstein*, the relation among rest mass m, linear momentum p and energy E is $E^2 = m^2c^4 + p^2c^2$ where $p = mv/\surd[1 - (v/c)^2]$. As also presented in appendix C, the result for a collision between a photon with wavelength λ and a mass m at rest is $\lambda' - \lambda = [h/(mc)](1 - \cos\theta)$ where λ' is the wavelength of the scattered photon and θ is the angle, relative to the incident photon, of scattering as shown in figure 2.4. The quantity $h/(mc)$ is called the Compton wavelength and has a value of 2.43×10^{-12} m = 0.00243 nm for electrons; so for backscattered photons ($\theta = 180°$, $\cos\theta = -1$) the change in wavelength of the radiation is twice the Compton wavelength, 0.00486 nm. Now, since the wavelengths of light are hundreds of nanometers, it would be extremely difficult to notice this small a change. X-rays, however, have wavelengths on the order

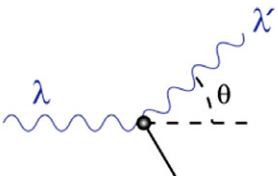

Figure 2.4. Geometry for Compton scattering.

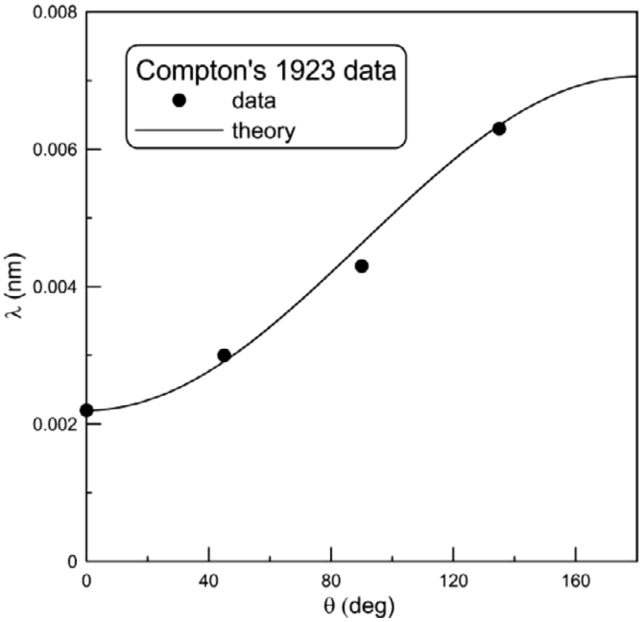

Figure 2.5. Compton scattering of 0.56 MeV photons.

of 0.01–10 nm, so the change in wavelength should be observable, particularly for 'hard' x-rays which are below 0.2 nm. Even better would be γ-rays with yet shorter wavelengths. (As we shall see in later chapters, x-rays come from atoms and γ-rays come from nuclei.) Compton used x-rays of about 0.07 nm and γ-rays of about 0.0022 nm. The results of his γ-ray experiment are shown in figure 2.5.

Taking stock, three experiments have established that light is composed of particles called photons. But, how can that be? The wave nature of electromagnetic radiation had been firmly established by numerous experiments, not to mention the Maxwellian theory of electromagnetism, over more than a century. The situation seemed to be:

- if you look for a wave, you will find a wave, and
- if you look for a particle, you will find a particle.

In other words, light is *both a particle and a wave*. This is different from *either a particle or a wave.* The phenomenon is a coexistence of both possibilities. This has come to be referred to as the *wave–particle duality*. This duality is very much at the heart of what would become quantum mechanics.

2.6 de Broglie's hypothesis

It would now be a logical question to ask whether, since it has been shown that light is *both* a wave and a particle, why wouldn't we expect any particle to behave like a wave? This was the question addressed by a young French student, Louis de Broglie (1892–1987), in his 1924 doctoral dissertation. The idea is pretty simple. Because photons have energy $E = hf = pc$, the linear momentum p may be written $p = hf/c$.

But, since $f/c = \lambda$, de Broglie suggested that any particle with momentum p might also be a wave with wavelength $\lambda = h/p$. Today, this is called the de Broglie hypothesis and the wavelength of a particle is called the de Broglie wavelength.

Question: I have learned that every subatomic particle behaves like a wave and that it can be described using a wave equation, however, what exactly does it mean when they say waves? I know photons travel in waves, but how do protons, electrons, and other particles exhibit wave behavior?
Answer: It is one of the tenets of quantum mechanics that any particle is also a wave. Look for a particle, you will find particle-like properties, look for a wave, you will find wave-like properties. This was first proposed by Louis de Broglie in 1924 and he won the Nobel Prize for his hypothesis in 1929. It has subsequently been verified innumerable times in experiments. For one example of diffraction of protons by a sphere, see below.

Question: Is proton diffraction possible?
Answer: Any particle, including a proton, will behave like a wave if you look for it, and diffraction is possible for all waves. In fact, I can give you an example of proton diffraction which I myself measured. What is shown in figure 2.6 is the differential cross section for 800 MeV protons elastically scattered from the nucleus ^{90}Zr plotted as a function of the angle (in degrees) where the protons were observed. Differential cross section is, essentially, the probability that the proton (wave) will scatter (diffract) to some angle. This is very much like the diffraction pattern you would observe for visible light striking a sphere—diffraction maxima and minima. In fact, you can qualitatively understand this diffraction pattern if you calculate the wavelength of the protons, $\lambda = h/p \approx 10^{-15}$ m, and approximate the positions of the maxima by the double slit relation $n\lambda = d \sin\theta$ where we

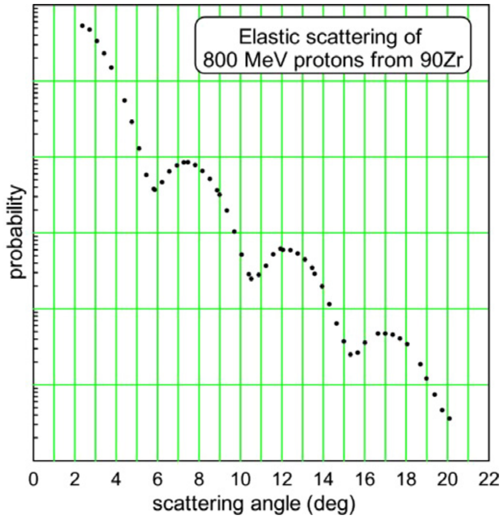

Figure 2.6. Scattering of 800 MeV protons from ^{90}Zr.

take d to be the diameter of the nucleus. Taking the two consecutive maxima at about 7.5° and 12.5°, $\lambda \approx d(\sin 12.5° - \sin 7.5°) = 0.086d = 10^{-15}$ m. So, the diameter of the ^{90}Zr nucleus would be about 11.6×10^{-15} m = 11.6 fm. As a check, the diameter of a nucleus with atomic weight A is well approximated as $d \approx 2 \times 1.25 \times A^{1/3}$ fm, so for $A = 90$, $d \approx 11.2$ fm, pretty good agreement for such a rough calculation. The experiment was performed at the Los Alamos Meson Physics Facility (LAMPF).

The example above exemplifies the difficulty faced by physicists trying to experimentally verify de Broglie's hypothesis—particle waves often have incredibly short wavelengths. The 800 MeV protons in the example have a wavelength of about 1 fm which is smaller than the size of the atomic nucleus. Just try making a double slit with that spacing! Electrons, with their much smaller mass, have longer wavelengths for comparable velocities, but the electrons available in the 1920s had energies of up to a few hundred electron-volts with wavelengths on the order of 10^{-10} m. But that is manageable because the spacing between atoms in a solid is about this size and in 1927 the definitive experiment was performed by Davisson and Germer who were able to demonstrate diffraction from a nickel crystal. The detailed analysis of reflection diffraction from the three-dimensional array of atoms in the crystal is not important for our purposes; suffice it to say that, again, diffraction similar to the Young double-slit experiment was responsible for verifying the wave nature of electrons. Davisson and de Broglie were awarded the Nobel Prize in 1929.

This may be a good time to ask the perennial question—what is the good of all this 'pure science research'? Well, as the Q&A below illustrates, one of the most important instruments of the 20th century is the electron microscope. As explained below, optical microscopes are limited in what they can see because of diffraction. However, electrons behaving like waves have much shorter wavelengths than light and can therefore 'see' much smaller objects. An electron microscope uses magnets to manipulate the rays of electrons in the same way that lenses and mirrors are used to manipulate rays of light.

Question: Why do electron microscopes get better pictures than regular light microscopes?

Answer: The problem with optical microscopes is that you cannot look at anything which is comparable to or smaller than the wavelength of the light. Visible light has wavelengths of a few hundred nanometers, ~6×10^{-7} m, so what you are looking at has to be bigger than that, a few microns maybe. The reason is diffraction, light does not just travel in nice straight lines when obstructions or apertures get to be on the order of the wavelength, e.g. light can be bent around a corner. If light does not go in nice straight lines, geometrical optics doesn't work. On the other hand, we know that particles like electrons can behave like waves and their wavelengths are inversely proportional to their momentum. So, if we make an electron go fast enough it will have a wavelength much smaller than light and therefore let us 'see' much smaller things.

2.7 What's waving?

It is now into the 1920s and evidence seems irrefutable that particles and waves are two sides of the same coin.

> **Question:** How is it possible that light can be both a particle and a wave? I need to know, it's ruining my whole life!
> **Answer:** Ruining your whole life? How is it possible that my tie can be both red and green? That is just the way it is. I know that this is an unsatisfying answer, but science is based on measurements (experiments) and if you study light and look for a wave, you will find one but if you look for a particle you will find that also. This is called the wave–particle duality and it is, essentially, a statement of experimental fact. And you can do experiments which unambiguously see both possibilities in the same experiment. The best known example is to do a double slit experiment with very low intensity. If the intensity is so low that there is, say only one photon per minute passing through, then it has to pass through one slit or the other, right? Wrong—you still obtain an interference pattern. So, you say, light is therefore a wave, not a particle. But, imagine that the screen detects the light using the photoelectric effect which cannot be understood unless you treat the light as a collection of photons. This detector will work perfectly well in displaying the interference pattern. This duality is not unique to light: any particle will exhibit wave properties; e.g. that is how an electron microscope works, by exploiting the wave properties of electrons. Maybe I shouldn't have told you this since it may compound your life-ruining distress! Wave–particle duality is a reality of nature, that is all.

We are faced with the same question which 19th century physicists were faced with when it became clear that light was some kind of wave—what is doing the waving? Without going into detail, it turns out that by analogy with classical mechanics one may write an equation the solutions to which are called the wave functions. This equation is called the Schrödinger equation and the details become quite mathematical, so they are included only in appendix E. But the solutions of this equation for some particular physical system (say an atom or a Planck oscillator), are called the wave functions of the system. The wave function, on its own, really has no physical meaning. But if you take the square of the wave function, it tells you the probability of finding the particle at each location in space. The Schrödinger equation and its detailed solution are beyond the scope of this book, but we can look at a couple of simple cases to obtain the flavor of the wave function and its meaning. The real world is three-dimensional, so the most general wave function (squared) will give you the probability of finding the particle in some small volume around some point in space. It is much easier to understand the nature of the wave function if we let our space be one-dimensional. And such a space is not unphysical, just constrained; one simple example would be a particle confined to move along a straight line, for example a bead on a wire.

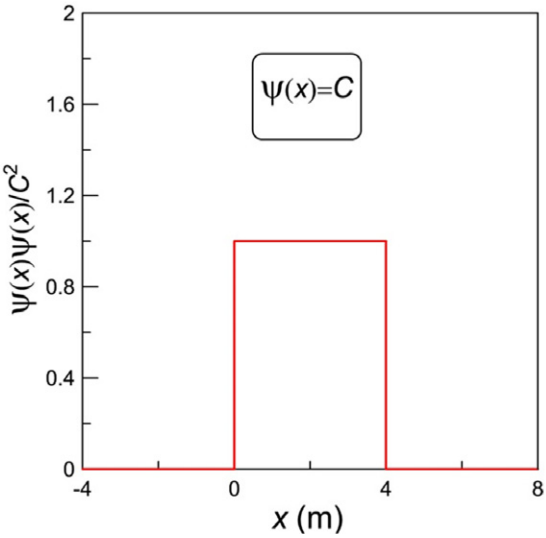

Figure 2.7. A simple one-dimensional probability distribution.

Suppose that the wave function $\psi(x)$ has the simple form of a constant $\psi(x) = C$ between the two points $x = 0$ and $x = 4$ m and is zero everywhere else. Just imagine a bead on a wire stretched between two walls separated by 4 m. As noted above, what is of interest is the square of the wave function $\psi(x) \times \psi(x) = C^2$; since this is the probability distribution, this means that it is equally probable to find the bead anywhere on the wire and it will never be found outside the walls. Then the square of the wave function is C^2. There is no chance that you will find the particle somewhere other than between 0 and 4 m. I have plotted $\psi(x) \times \psi(x)/C^2$ in figure 2.7.

A little common sense now will allow us to determine what C has to be. The probability that the particle be between the walls is 1; the probability that it is between one end and the middle is $\frac{1}{2}$; the probability that it be in any 1 m distance along the wire is $\frac{1}{4}$; etc. The preceding should tell you that what determines the probability P of finding it in some region Δx along the wire is the area under $\psi(x) \times \psi(x)$ in that region, $P = \psi(x) \times \psi(x)\Delta x$. Therefore, for this example, $C^2 = \frac{1}{4}$ or $C = \frac{1}{2}$. (To check this, $\psi(x) \cdot \psi(x)\Delta x = \frac{1}{2} \times \frac{1}{2} \times 4 = 1$ if $\Delta x = 4$ m.) This process is called normalization; requiring the total area to be equal to 1 allows us to interpret the area as a probability.

The same reasoning as above applies to any wave function, not just a constant—the total area under $\psi(x) \times \psi(x)$ must be 1. If you know integral calculus, this is written as $\int_{-\infty}^{\infty} \psi(x) \times \psi(x)\mathrm{d}x = 1$. An example of a more realistic wave function (called a Gaussian function) is shown in figure 2.8. Here the particle is most likely to be near $x = 0$ and the likelihood of finding it gradually decreases to near zero beyond $|x = 2|$.

Now that we have an idea of what a wave function is, we can discuss one of the main consequences of wave–particle duality, the Heisenberg uncertainty principle.

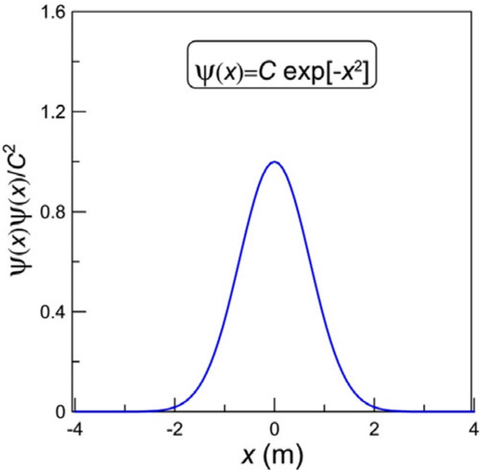

Figure 2.8. A more realistic probability distribution.

2.8 The Heisenberg uncertainty principle

2.8.1 Introduction

In figure 2.8, the particle being described by the wave function is most likely to be found between $x = -2$ and $x = 2$. This statement alone leads you to appreciate that you do not know precisely where it is, only a probability of where you are likely to find it. But this uncertainty is not the famous uncertainty principle you have heard about for the following reason. When you make a measurement of where a particle is you are in fact causing it to have a wave function which is consistent with your measurement. But as you make better and better measurements you cause the wave function to become smaller and smaller until you finally determine the position to any arbitrarily well-determined value. This turns out not to be so simple because there are really two things you do not know about a particle, its position and its speed (i.e. its momentum). The reality is that the more accurately you know one of these, the less accurately you know the other—you cannot simultaneously know both the momentum and position of a particle to arbitrary precision. Understanding this is what this section is about.

> **Question:** I am really confused about what the Heisenberg principle actually states. Why can't the position and momentum be measured together? Can't we measure the position and momentum at any particular instant of time since physics can do anything?
> **Answer:** Physics can do anything? Unfortunately, nature has put limits on how precisely certain things can be determined. The underlying reason for the uncertainty principle is the fact that there is really no such thing as a particle. Anything which you might think of as a particle has a dual reality, it is also a wave. In what way, for example, is an electron like a wave? Think of a very long wave with some frequency f. It turns out that the momentum of the wave depends

on f. The 'position' of this wave train is uncertain, you will admit because it has some length where it exists. Now I said earlier that the wave has a frequency f, but in reality, it has a well-defined frequency only if the wave train is infinitely long. If you want to write an expression for the finite wave, you find that it is really made up of an infinite superposition of frequencies; this is called a Fourier transform, decomposing some wave into simple sinusoidal functions. The most important contributor is the momentum corresponding to f, but there is a whole distribution of others. So, you see that the wave has uncertainty in both position and momentum. As I said above, infinite uncertainty in position will result in zero uncertainty in momentum. Similarly, if you try to pin down the position by making the wave train shorter and shorter, you find that the momentum distribution becomes larger and larger.

The essence of what is at the root of the uncertainty principle is contained in the answer to this question. To fully appreciate the connection between frequency and momentum, and how momentum and position are intertwined, we will digress briefly to talk about the mathematics involved in Fourier analysis of wave forms. It will not be as bad as it sounds.

2.8.2 The timbre of musical instruments

Have you ever wondered why you can tell the difference between the sounds of two musical instruments playing exactly the same note? Musicians call the quality which allows such a distinction timbre.

Question: A chromatic musical instrument can be used to play any key, but does it play them all equally well? In other words, will it sound better in certain keys? I have read a lot about the harmonic series and fundamental notes. I know that individual notes will have slightly varying timbres (harmonic series). But will the fundamental note have the richest harmonics?
Answer: I guess, first of all, we should say that 'sound better' is a subjective thing. The physics of musical instruments is a field unto itself and I certainly cannot cover it in a concise answer which is the *modus operandi* of this site. A few things are worth pointing out, though. First, terminology: the fundamental frequency is the lowest frequency present in the note, other frequencies present are called overtones, sometimes overtones are harmonics which are integer multiples of the fundamental frequency. Why does the same note played on a piano and a violin sound so different? It is determined by the timbre of the note played on each instrument and this is, essentially, the *relative* amplitudes of the fundamental and all the overtones produced by the instrument. The amazing thing is that many musical sounds which your brain perceives as some note contain much more of the overtones than of the fundamental, the frequency of the note which your brain is hearing. In fact, if you remove the fundamental entirely from the spectrum, your brain will still 'hear' the note as the fundamental. That is why tiny speakers which cannot put out low frequencies, say $f = 20$ Hz, still sound like they have a

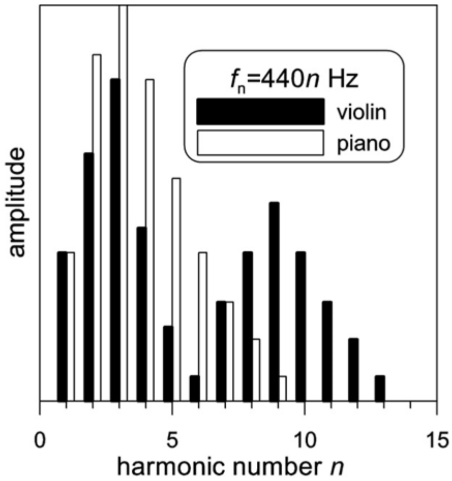

Figure 2.9. Fourier components for a violin and a piano playing A3.

bass response. As an example, figure 2.9 shows the amplitudes of the overtones for a violin and a piano playing the note A3 (440 Hz). For these instruments playing this note, the overtones are all harmonics. Note that the violin has a much greater contribution from higher frequency overtones and a relatively damped mid-frequency response. Also note that both instruments have a fundamental amplitude smaller than some of the overtones. It becomes more complicated still if you consider time; if you look at the relative loudnesses of all the overtones, they change from when the note begins (say plucking a string) and when it ends (string stops vibrating). Finally, the overtone composition depends on how the instrument is played as well as the instrument itself.

Now, you are probably asking yourself, 'what on Earth do pianos and violins have to do with photons and quanta?' What Fourier analysis allows you to do is to represent any complex wave form (such as the sound wave from a musical instrument) as a sum of simple wave forms (sine functions). What we have done so far in this subsection, is show how a periodic function (one which identically repeats its shape after some period T forever) can be represented by a sum of overtones which are simple periodic functions of frequencies $nf = n/T$. In the next subsection we will generalize this to more localized (realistic) situations.

One more thing before we move on. For musical instruments it was useful to talk about time t and frequency f. However, for our purposes, it is more useful to look at how the wave looks in space, x, and decompose it in terms of simple wavelength λ; so, instead of decomposing some $\psi(t)$ in terms of frequencies f, we will decompose some $\psi(x)$ in terms of wavelengths λ.

2.8.3 Fourier transforms and the uncertainty principle

Suppose that we now have a function which we wish to decompose into Fourier components, but it does not seem to be periodic. Take, for example, the wave

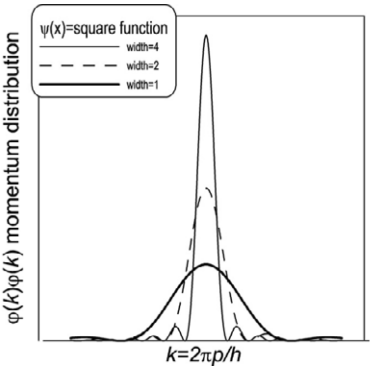

Figure 2.10. Fourier transforms of a square function for three widths.

function in figure 2.7, a square function which is localized between $x = 0$ m and $x = 4$ m. Is this a periodic function? It does not repeat itself and so we are inclined to say that it is not periodic. On the other hand, we could say that it has an infinite period, so perhaps Fourier series could be generalized to include the situation of infinite wavelength which is what the wavelength of this wave is. Without going into detail, this can be performed mathematically and the only difference between this and the Fourier series is that rather than just the harmonics being the overtones, all frequencies are the overtones. The resulting distribution of wavelengths, $\varphi(k) \times \varphi(k)$, (where $k = 2\pi/\lambda$ is called the wave number) contributing to make $\psi(x) \times \psi(x)$ of figure 2.7 is shown by the thin curve in figure 2.10, $\varphi(k) \times \varphi(k)$ where $k = 2\pi/\lambda$; also shown are calculations for $\varphi(k) \times \varphi(k)$ for reducing the width of $\psi(x) \times \psi(x)$ to 2 m (dashed) or 1 m (heavy line).

Now, what does it mean to reduce the width of $\psi(x) \times \psi(x)$? Recall that the wave function (squared) represents the likelihood of finding the particle somewhere; so reducing the width of $\psi(x) \times \psi(x)$ means that the particle is confined to a smaller region of space so that the uncertainty of the location of the particle Δx becomes smaller. But look what simultaneously happens to $\varphi(k) \times \varphi(k)$: as we reduce the width of the box, the width of the distribution of k becomes larger. If we recall from section 2.5 that $p = h/\lambda = hk/2\pi$, then $k = p/\hbar$ ($\hbar = h/(2\pi)$)—k is just a different way to represent p, so the more accurately you know x, the less accurately you know p (i.e. the speed). It should be evident that this cuts the other way as well—the more accurately you know p, the less accurately you know x. This is the famous Heisenberg uncertainty principle which is the inescapable consequence of a particle having wave-like properties. It is tricky to make a quantitative equation expressing this principle because it requires a quantitative definition of uncertainty, e.g. how do you measure Δp in figure 2.10? It is generally expressed as an inequality, $\Delta x\, \Delta p \geqslant \hbar$. Note that since $\hbar \approx 10^{-34}$ J · s is so small, you will never see the consequences in everyday life.

One final thing. Momentum and position are called conjugate variables. There are other pairs of measureable quantities which are also conjugate and also have uncertainty principles associated with them. The most important such pair for the purposes of this book is energy E and time t, $\Delta E \Delta t \geqslant \hbar$. The following Q&As illustrate this version of the uncertainty principle.

Question: How does vacuum fluctuation cause the formation of a particle–antiparticle pair in a vacuum?
Question: I read an article about quantum mechanics and it mentioned particles 'popping into existence'. Is the assumption that the particles are popping into existence from another invisible/unknown state of energy which exists, or is the claim that they are appearing out of 'absolute nothingness' and violating the law of conservation of energy? If the currently held belief is that the particles are actually appearing out of pure nothingness, how is it being proven that there is nothing there in the first place? I'm not a scientist obviously but I would have thought that absolute nothingness could not possibly change state into matter, and that there must be a real and invisible force already present which is temporarily changing state into those particles. Surely there must have been some energy somewhere.
Answer: The answer to the first question is that vacuum fluctuation is the scientific term for the creation of a particle–antiparticle pair. The second question gets more into the physics of this kind of pair production. The conservation of energy is one of the most important laws of physics, however, the Heisenberg uncertainty principle allows it to be broken provided that it is violated for a short enough time. $\Delta E \Delta t \approx \hbar$, where ΔE is the amount by which conservation is violated, Δt is the time the violation lasts, and $\hbar \approx 6.6 \times 10^{-13}$ keV · s where keV (kiloelectron-volt) is a unit of energy. The mass energy of an electron–positron pair is about 1000 keV, so if they are created out of nothing, they may exist for $\Delta t \approx 6.6 \times 10^{-16}$ s and then annihilate back to nothing. A proton–antiproton pair has 2000 times more energy and so could last for only about $\Delta t \approx 3.3 \times 10^{-19}$ s. So, there is no such thing as 'pure nothingness' as you put it, but you will never actually find anything there because anything which 'pops into existence' pops right back out again in almost no time. Such processes are called virtual pair production.

Question: Please can you explain to me the energy–time uncertainty principle and what are ΔE and Δt in it?
Answer: Generally it is pretty easy to visualize what the uncertainty principle involving position and momentum means—you cannot simultaneously know the position and momentum of a particle to arbitrary accuracy. However, there are other pairs of conjugate variables for which the uncertainty principle occurs, two examples are angle–angular momentum and energy–time. The energy–time uncertainty principle is a little harder to get a grasp of, as you have found. There are two examples I can think of which are physical consequences of $\Delta E \Delta t \sim h$.

- As you know, one of the truths of classical physics is that the total energy of an isolated system must be conserved. However, in quantum mechanics it is perfectly OK for energy conservation to be violated, as long as you do it for a very short time. So, suppose a photon with energy E is suddenly emitted from a charged particle. This violates energy conservation. But, if that photon disappears, is reabsorbed or absorbed by some other charged particle in a time on the order of $\Delta t \sim h/E$, that is OK. This gives rise to the so-called vacuum polarization where particles pop into and out of existence even in completely empty space, so we do not really think of a vacuum as containing nothing.

- Another way to look at $\Delta E \Delta t \sim h$ is that to know an energy absolutely perfectly, the measurement will take forever. So, the ground state of an atom has a well-defined energy because it never changes and is 'available' forever. So, apart from experimental uncertainties, you can measure the energy of a stable atom perfectly accurately. However, the excited state on an atom does not live forever, it decays and the decay is characterized by a half-life, $\tau_{½}$. Therefore, if you measure the energy of such a state by measuring the energy of the emitted photon, after many measurements you will find a spread of energies $\Delta E = \Gamma \sim h/\tau_{½}$. Because an excited atom does not last forever, that energy state does not have a specific energy and the spectral lines are not really perfectly sharp. Very short-lived states have very noticeable widths. Measuring the widths of states is often a more accurate way of measuring half-lives.

Chapter 3

Atoms, oh my!

3.1 Atomic sizes and masses

The atomic composition of matter is one of the oldest conceptualizations in the human quest to understand the world. The Greek philosopher Democritus proposed that matter was composed of small, indivisible units he called atoms (from the Greek *ατομον* meaning uncuttable) around the 5th century BCE. The idea was also recorded by Indian philosophers around the 6th century BCE. These were philosophical ideas, not based on any measurements, and could therefore say nothing about the size, mass, or structure of atoms. In the 18th century, when chemistry was first being studied seriously, the idea of atomism was resurrected as the only plausible foundation for chemistry. By the early 19th century chemists had pretty good quantitative measures of the size and mass of atoms.

> **Question:** No one has ever seen the atom because it is really very, very, very small, then how is its size known? I mean, how do we know that an atom has a radius of 10^{-10} m?
> **Answer:** The size of an atom has been known for a long time, since the early 19th century. Avogadro's number is $N_A \approx 6 \times 10^{23}$ atoms per mole and tells you how many atoms there are in a mole of a substance; density is easily measured. For example, carbon has an atomic number of 12 so a mole has a mass of 12 g. The density of carbon is about $2\,\mathrm{g\,cm^{-3}}$, so one mole of carbon occupies a volume of about $12/2 = 6\,\mathrm{cm}^3$. Therefore the volume occupied by one carbon atom is $V \approx 6/6 \times 10^{23} = 10^{-23}\,\mathrm{cm}^3$, so the size (diameter) of a carbon atom is about $\sqrt[3]{10^{-23}} \approx 2 \times 10^{-8}\,\mathrm{cm} = 2 \times 10^{-10}\,\mathrm{m}$. Of course, today individual atoms can be 'imaged', just not in the usual optical way; see the question below and figure 3.1.

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Figure 3.1. Individually imaged atoms.

> **Question:** Can we literally see an atom given today's technology?
> **Answer:** Depends on what you mean by 'see'. If you mean seeing light from the atom forming an image of it in a microscope, like we observe bacteria for example, then no. The reason is that an atom is much smaller than the wavelength of visible light and geometrical optics does not allow that. But individual atoms can be imaged with electron microscopes or things called atomic force microscopes and scanning tunneling microscopes. Google those to see how they work. Atoms can be manipulated too. Figure 3.1 shows a famous IBM image of individual atoms pushed around to spell IBM.

In the first question above, the size of a carbon atom is calculated. It turns out that, roughly speaking, all atoms have about the same size. To demonstrate this, consider lead which has an atomic weight of 207 and a density of $11.3\,\mathrm{g\,cm^{-3}}$. The volume of one mole is $207/11.3 = 18.3\,\mathrm{cm}^3$, so the volume of one atom is $18.3/6 \times 10^{23} = 3.1 \times 10^{23}\,\mathrm{cm}^3$ and the size is about $\sqrt[3]{3.1 \times 10^{23}} = 3 \times 10^{-8}\,\mathrm{cm}$. This is only about 50% larger than carbon which is 200 times lighter than lead.

Atomic masses, on the other hand, vary proportionally to atomic weights. From the carbon results above the mass on one atomic weight unit can be determined by simply multiplying the volume of one atom by the density and dividing by 12: $[2\,\mathrm{g\,cm^{-3}}][10^{-23}\,\mathrm{cm}^3]/12 = 1.7 \times 10^{-24}\,\mathrm{g} = 1.7 \times 10^{-27}\,\mathrm{kg}$.

These sizes and masses are so small, they are very difficult to visualize. There are more atoms in your pencil than there are stars in the entire known Universe.

> **Question:** I want to give my students a relative understanding of the size of an atom. If a bowling ball were enlarged to the size of the entire Earth, (and its atoms enlarged proportionately), how big would just one of its atoms be? The size of a house? A beach ball?
> **Answer:** The radius of a bowling ball is about 0.1 m and the radius of the Earth is about 6×10^6 m. So the magnification is about 6×10^7. The radius of an atom is about 10^{-10} m, so the magnified atom would have a radius of about $6 \times 10^7 \times 10^{-10}\,\mathrm{m} = 6 \times 10^{-3}\,\mathrm{m} = 6\,\mathrm{mm}$, about a quarter of an inch.

As you can see, 19th century scientists had a quite good knowledge of the sizes and masses of atoms, mainly through the efforts of chemists. Also from chemistry there was much empirical knowledge of molecules and chemical reactions. What they did not have was any knowledge of the composition or structure of atoms. It should come as no surprise to you that a detailed description of atomic structure would have to wait until the 20th century when the ideas of quantum physics were developed (chapter 2); you should, by now, always be skeptical whether classical physics will be applicable when you are outside the realm of your experience—very small, very fast, very large.

3.2 Atomic spectra

As discussed in earlier chapters, Newton observed the spectrum of radiation from the Sun to be a continuous spectrum and later it was found that the distribution of intensity of the solar spectrum is very well described as black-body radiation. However, this turns out to be not exactly true. In 1802 William Wollaston observed very sharp dark lines in the solar spectrum. Subsequently, in 1819, Joseph von Fraunhofer observed hundreds of dark lines; a few of the most prominent lines are shown in figure 3.2. Today these are known as absorption lines; apparently the Sun is radiating like a black body but some unknown agent is absorbing certain discrete wavelengths.

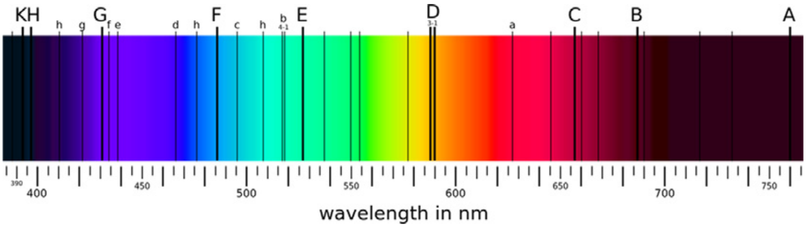

Figure 3.2. Dark lines in the solar spectrum.

In 1826, John Herschel discovered that if you heated and vaporized any element, it emits not a continuous spectrum but rather only discrete lines. And the pattern of these lines were distinctive for each element, providing chemists with a much more convenient means of determining the composition of materials than more traditional chemical analysis. But more importantly the discovery of discrete spectra would eventually be the doorway to discovering the structure of atoms. The spectrum of hydrogen is shown in figure 3.3. Now, if this spectrum is compared with the absorption spectrum in figure 3.2, the same wavelengths appear in both. And, other emission lines can also be identified; for example, the two lines labeled D in figure 3.2 are of the identical wavelengths as the two brightest lines in the spectrum of the element sodium. Apparently, then, atoms have discrete energy levels, they are quantized just like Planck's oscillators are but in some much more complicated way. (Recall that a Planck oscillator emits only one single wavelength.) An atom emits a photon when

Figure 3.3. The visible line spectrum of hydrogen.

making a transition from one energy state to another lower state, but it can also make the same transition in reverse by absorbing a photon of the same wavelength.

In 1885 Johann Balmer discovered empirically that the hydrogen lines in the visible part of the spectrum have a quite simple form, $\lambda = Bn^2/(n^2 - m^2)$ where $B = 364.5$ nm, $m = 2$ and $n > 2$. Subsequently additional lines in the ultraviolet and infrared parts of the hydrogen spectrum were observed and found to be well described for other choices of m. Now, a model of the structure of atoms was needed which would explain the discrete spectra of atoms. There was one major barrier to doing this—nobody had any idea what atoms were made of! The best idea anyone had come up with was that, since hydrogen was the lightest atom, all others must be made up of clusters of hydrogen atoms.

3.3 The discovery of the electron

By the middle of the 19th century devices called cathode-ray tubes had been discovered. They were essentially just an evacuated glass tube with a metal plate at each end. When a high voltage was applied across the plates, some kind of ‘ray’ was observed (see figure 3.4). In 1897 J J Thomson was the first to demonstrate that these rays had a negative electric charge since he could deflect them with electric or magnetic fields. By measuring the deflection he was able to determine the ratio of the charge to the mass of the particles, $q/m \approx 1.8 \times 10^{11}$ C kg^{-1}. At first it was assumed that the particles were atoms which had somehow acquired a negative charge, but, given his measured value of q/m, the charge would have had to be huge if m were as heavy as an atom. He assigned a student, John Townsend, to try to determine the electric charge of the particles. This turned out to be very difficult, but he was able to do an order-of-magnitude determination of $q \approx 10^{-19}$ C. This then resulted in a mass approximately $m \approx 5.6 \times 10^{-31}$ kg, about 3000 times smaller than the mass of a hydrogen atom. Thomson was the first scientist to discover an elementary particle, the electron. Later a more accurate determination of the charge was made by Robert Millikan, $q = 1.6 \times 10^{-19}$ C, so $m = 9 \times 10^{-31}$ kg.

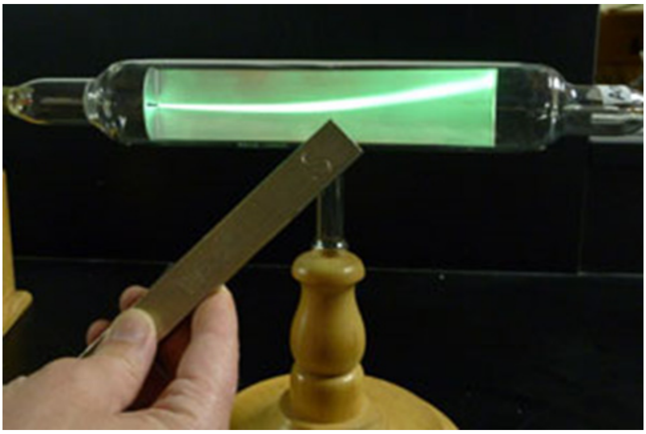

Figure 3.4. Cathode ray deflected by a magnet.

> **Question:** I would like to know what impact did Millikan's oil drop experiment have on science during and before 1920. Why is the electron charge so important? What is it used for?
> **Answer:** This is a strange question! I believe that understanding the world around us in as much detail as we can is required by the human spirit. If you did not know the electron charge you could have no atomic physics. Experiments like that done by J J Thomson were able to measure the ratio of the charge to the mass, but to obtain either you had to measure one independently which is what Millikan's experiment did. So you could say that knowing the charge gives you the mass and knowing the mass of something is important in physics. The electron charge is used, like many other fundamental constants, for understanding the universe; what could be more important than that?

Since it was known that atoms are electrically neutral, there must be some positive charge in there as well. Thomson proposed that the structure of an atom was a sphere of evenly distributed mass and positive charge containing most of the mass of the atom (since an electron was about $1.7 \times 10^{-27}/9 \times 10^{-31} = 1900$ times less massive than a hydrogen atom). The electrons were then envisioned to be distributed around inside the atom and at rest—at rest because, as we learned in chapter 1, accelerating charges radiate and they would radiate away their kinetic energy. This has become known as Thomson's 'plum pudding model'. However, nobody could figure out how such a model could predict the atomic emission spectral lines.

3.4 Ernest Rutherford and the 'nuclear model'

Suppose that you had a wall in front of you and it was made of some kind of positive charge and mass distributed in the volume of the wall but you did not know how it was distributed. Embedded in this wall were many electrons, in fact just the right number so that the net charge of the wall was zero; also they would presumably be more or less uniformly distributed. How might you find out how the positive charge and almost all the mass of this wall was distributed? If you had a 'gun' which shot positively charged bullets which had a mass much greater than the mass of an electron and observed what happened when you shot them at the wall, you could deduce the positive charge and mass distribution of the wall. A series of experiments was performed by Ernest Rutherford, Hans Geiger and Ernest Marsden over the period 1908–13. Their 'wall' was a very thin gold foil and their 'gun' was a radioactive radium source of energetic (several MeV) α-particles. An α-particle is a helium atom stripped of its two electrons which has a mass about 8000 times heavier than an electron and a positive charge of magnitude twice the electron charge.

> **Question:** Suppose all the α-particles in Rutherford's experiment went straight through the gold foil with absolutely no deflections. What would this imply about the structure of the atom?

Answer: It would mean that the α-particles had no (or at least very small) interactions as they passed through the foil. The electrons in the foil would have virtually no effect on the α-particles because of the disparity in masses (think of throwing bowling balls at BBs). Because the foil is electrically neutral, the α-particles would feel no force when approaching or leaving the foil. According to Thomson's plum pudding model, the charge density of positive charge would be approximately uniform inside the foil. Once inside, the α-particle would see the same amount of charge regardless of which direction it looked and so the net force on it would be zero and it would pass through undeflected. So your imagined result of the Rutherford experiment would support the plum pudding model.

An interesting question, but not what was observed. Instead α-particles were observed to scatter, in fact some scattered backwards with angles greater than 90°. The results of Geiger and Marsden's experiments are shown in figure 3.5. (Note that this is a logarithmic scale for rate and drops more than a factor of 4000 over the range of the data.) Rutherford now made a radical guess—what would the prediction be if he assumed that all the mass and positive charge were simply a point in the center of the atom. As you can see, this assumption fits the data nearly perfectly. Of course, the nucleus is not of zero size, but the α-particle did not have enough energy to get close enough to 'touch' the nucleus; the data allowed an upper limit on the radius R of the nucleus, $R < 30 \times 10^{15}\,\mathrm{m} = 30\,\mathrm{fm}$. Today we know that the radius of a gold nucleus is approximately 7.3 fm. By doing experiments on lighter nuclei like aluminum, Geiger and Marsden were able to obtain estimates of radii by observing the angle at which the data failed to be adequately predicted by Rutherford's model.

Question: I've read that if an atomic nucleus was scaled up to the size of a period in size twelve font, the nearest orbiting electron would be 3 m away. I've also heard that a nucleus the size of a baseball at 'home plate' would have its nearest electrons orbiting at 2nd base. Is there a specific ratio involved for 'size of nucleus':'distance of electrons?'

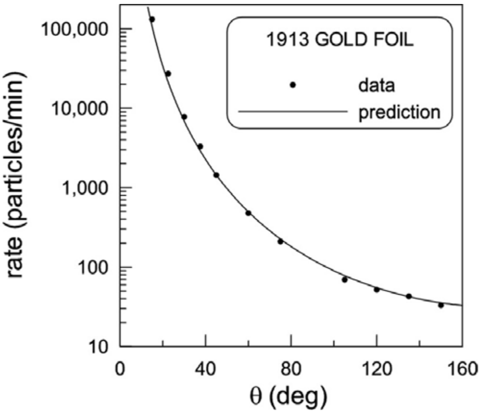

Figure 3.5. Geiger and Marsden's 1913 data and Rutherford's prediction for α-particle scattering from gold.

Answer: The nearest orbiting electron is not a very good yardstick to get a feeling for nuclear sizes because it depends on what atom you are talking about. Lead will have a much smaller 'nearest orbit' than helium will. The reason is that the electric force on the innermost electrons is much stronger because the charge of the heavier nucleus is much larger. What is more meaningful is to compare the size of the nucleus with the size of the atom (which quantifies the largest rather than smallest orbits). An atom has a diameter on the order of 10^{-10} m whereas the nucleus has a diameter on the order of 10^{-14} m. So the size ratio is ~1:10 000. If the atom is the size of a football field (100 m) the nucleus is the size of $100 \times 10^{-4}\,\mathrm{m} = 1$ cm, about the size of a gumball.

So, the nucleus is incredibly small. When you see a depiction of an atom like figure 3.6, the nucleus is very poorly represented. Indeed, the electrons are also poorly depicted as we will see later in this chapter. Regarding what the nucleus is made of, that is a question we will address in chapter 4.

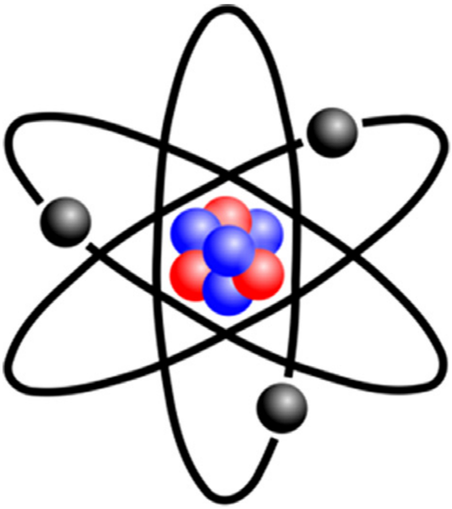

Figure 3.6. A poor depiction of an atom.

3.5 The Bohr model of the hydrogen atom

In this chapter there has been no 'new physics' thus far. There were hints in atomic spectra that energies of atoms are quantized like Planck oscillators were found to be, but there has been no quantized model thus far. And, with the results of the Rutherford experiment, making a model is very problematical because, with all the positive charge confined to one tiny volume, it becomes impossible using classical electromagnetism to have a static atom—electrons simply cannot be at rest and be in equilibrium. And the electrons cannot move within the atom without accelerating, so they would radiate away all their energy and collapse into the nucleus. Time for someone with a fresh idea!

In 1913 the Danish physicist Niels Bohr proposed a model for the hydrogen atom. The mathematical details are presented in appendix F; the main assumptions and results are:

- The electron, with charge $-e$ and mass m, moves in circular orbits of radius r around the nucleus which is approximated as having very large mass and charge $+e$.
- The energies of the orbits are calculated using usual classical mechanics, $E = -ke^2/(2r)$; the negative sign is relative to the energy of an electron at rest infinitely far away from the nucleus being zero and indicates that this is a bound system.
- Now the energy is quantized by requiring that only orbits with quantized angular momentum $L = mvr = n\hbar$ are allowed.
- Allowed orbits have radii given by $r_n = n^2\hbar^2/(kme^2)$ where $k = 1/(4\pi\varepsilon_0)$. (Remember ε_0, the permittivity of free space?)
- Allowed orbits have energies given by $E_n = -mk^2e^4/(2n^2\hbar^2) \equiv -R/n^2$.
- Now, when the atom drops from the n level to the n' level, a photon with frequency $f_{nn'} = (E_n - E_{n'})/h = c/\lambda_{nn'}$ is emitted.
- Solving for the wavelength, $\lambda_{nn'} = (n^2n'^2hc)/[R[(n^2 - n'^2)]$ with $n > n'$.

Essentially what Bohr did was simply postulate that if the electron was in one of the ‘allowed’ orbits it would not radiate except to drop down to a lower allowed orbit, thereby having a line spectrum as observed experimentally. Also, letting $n' = 2$ in the expression for $\lambda_{nn'}$ and evaluating the numerical factor leads to $\lambda_{n2} = (4n^2hc/R)/(n^2 - 2^2) = Bn^2/(n^2 - 2^2)$ where $B = 364$ nm, exactly the Balmer series expression described in section 3.2.

Although the model is certainly ad hoc, and certainly most of it is incorrect from a more modern perspective, it is getting something right and puts the whole field

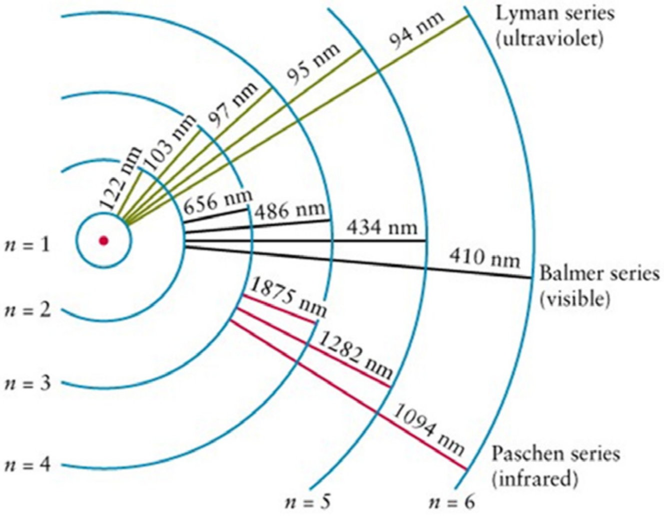

Figure 3.7. A picture of the Bohr atom showing three of the spectral series.

of atomic physics on a path to a more complete quantum description. Shown in figure 3.7 is a picture of the atom with the Balmer series depicted as the transitions to the first excited state, $n = 2$. Transitions to the ground state, $n = 1$, called the Lyman series, are all in the ultraviolet region of the spectrum; λ_{21} is called the Lyman-α line and plays an important role in the spectroscopy of stars. All other series are in the infrared and the Paschen series, with $n = 3$, is shown as one example.

There is one simple way for the Bohr atom to become a little more believable, as exemplified by the next question.

> **Question:** Could you explain why an electron in a stable orbit around a nucleus does not emit electromagnetic waves or photons. It only emits when the electron changes from a higher-energy orbit. In a stable orbit the electron still is moving around so why doesn't it create a changing E field and radiate?
> **Answer:** The glib answer is simply that that is the way nature works. Here we encounter an example of how we should not extrapolate the behavior of something from a regime with which we have experience (for example, accelerating electrons in a transmitting antenna) to regimes where we have no experience (for example, tiny atoms). At the scale of atoms, the wave-like properties of electrons become important and quantum mechanics must replace Newtonian mechanics; when you solve the problem using the correct mechanics, stable orbits occur naturally. I find the easiest way to understand this qualitatively is to note that in an atom a stable orbit is one in which the wavelength of the electron is just right to form a standing wave in its orbit as shown in figure 3.8 (de Broglie's picture of the Bohr atom).

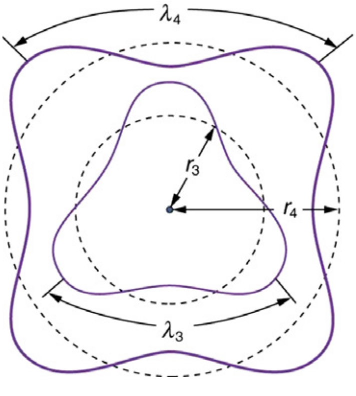

Figure 3.8. Standing de Broglie waves for the $n = 3$ and $n = 4$ states.

You should note that de Broglie made his hypothesis more than a decade later than when Bohr proposed his model, so this argument, while germane, could not have been made by Bohr. (See appendix H.) Bohr quantized angular momentum, $mvr = n\hbar = pr = nh/(2\pi)$. Rewriting, $2\pi r = nh/p = n\lambda_n$ where λ_n is the de Broglie wavelength of the electron in the nth orbit. What this says is that the circumference must be such that an integral number of wavelengths will just fit in. This is called a standing wave. Waves which do not satisfy this criterion will simply cancel themselves out and are therefore not allowed wave functions (zero probability). It is much like a guitar string which can only vibrate with frequencies which are multiples of the fundamental frequency. Figure 3.8 shows a cartoon of these waves for the $n = 3$ and $n = 4$ levels in the Bohr atom.

Although some additional work was carried out, generalizing Bohr's circular orbits to include elliptical orbits, called the Bohr–Sommerfeld model, the handwriting was on the wall when wave mechanics for particles became understood after de Broglie's work. The Bohr atom, while a very important step forward in understanding atomic structure, was naïve, simplistic and ad hoc. The next stage would require that Schrödinger's equation be solved for the hydrogen atom.

3.6 The modern atomic model

3.6.1 Solving Schrödinger's equation for hydrogen

Next, Schrödinger's equation needs to be solved for the problem of a single electron experiencing a potential energy function $V(r) = -ke^2/r$ due to the presence of the heavy proton. This is well beyond the scope of this book, I will not even include the solution in an appendix. The equation to be solved is a three-dimensional second-order partial differential equation (see appendix E). I will give an overview of the results. Be wary, this is a little tedious, but you just need (for now) to understand three quantum numbers which an electron has.

It is important to keep in mind the chronology of various developments. The Bohr model was proposed in 1913 but the Schrödinger equation was not proposed until 1925. So there was quite a bit of time before this correct understanding of the hydrogen atom was achieved. Certainly World War I interceded to slow progress.

The energies of the levels are exactly the same as the simple Bohr model, $E_n = -mk^2e^4/(2n^2\hbar^2)$, $n = 1, 2, 3\ldots$ The number n is called the principle quantum number. However, it no longer makes sense to talk about the 'orbits', neither their radii nor their velocities, because there is no well-defined distance from the proton to where the electron is. The radial part of the wave function is a continuum.

> **Question:** I often hear how we 'Don't know where the electron is'. What do these physicists mean when they don't know where the electron is? Isn't it orbiting the nucleus of an atom like all the pictures I've seen in science books and so on or are they referring to something completely different?
>
> **Answer:** The picture of electrons going around in neat little well-defined orbits is useful but only as a rough picture, not at all as an accurate description of an atom. I think the best way to think of an atom is that electrons are distributed around the volume near the nucleus, like a cloud. This cloud represents the probability of finding the electron at any location. If the cloud is dense, it is likely to find the electron there; if not, it is not so likely. The electron may be thought of as being wherever the cloud is, so you know a volume in which you will find it, but you cannot predict exactly where it is at any time. In figure 3.9 you can see where the 'orbit' is, where the electron cloud is most dense. Even if it did move in an orbit, you would have no way of knowing precisely where it was at any time.

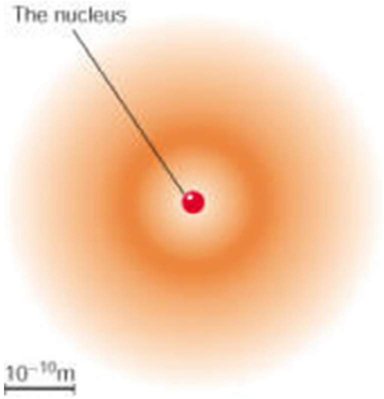

Figure 3.9. Electron cloud for the ground state of hydrogen.

Be sure to understand that figure 3.9 is a cutaway picture—the whole atom is like a fuzzy ball. The densest part of the cloud is a spherical shell which has a mean radius equal to the Bohr radius.

Recall that n also determined the angular momentum of the energy state in the Bohr model. This is no longer the case. A new quantum number ℓ, the orbital angular momentum quantum number now comes out of the solution of the Schrödinger equation. The orbital angular momentum L of a particular electron is $L = \hbar\sqrt{[\ell(\ell+1)]}$. Further, each energy level, characterized by the principle quantum number n, can have the values $\ell = n-1, n-2, n-3\ldots, 0$. So, the ground state has $\ell = 0$, the $n = 2$ state can have values $\ell = 0$ or $\ell = 1$, the $n = 3$ state can have values $\ell = 0, 1, 2$, etc.

Finally, the angular momentum is a vector and, it turns out, can only point in quantized directions. A new quantum number, often called the magnetic quantum number, m_ℓ emerges which can take on values from $-\ell$ to $+\ell$ in steps of 1, $-\ell \leqslant m \leqslant +\ell$; the physical meaning is the projection of L on a z-axis, $L_z = m_\ell \hbar$.

Reviewing the preceding two paragraphs, you will see that for each value of n there are $n+1$ values of ℓ and there are $2\ell+1$ values of m for each value of ℓ. The number N_n of distinctly different quantum states in an energy level n is therefore $N_1 = 1$, $N_2 = 1+3 = 4$, $N_3 = 1+3+5 = 9$, $N_4 = 1+3+5+7 = 16$, etc; in general, $N_n = n^2$. (Reader beware: this is still not quite the last word!)

Notation for specifying the quantum numbers is confusing. Many of the notational conventions are historical artifacts, names and letters which were assigned before everything was understood.

Question: What is the difference between ℓ and L in atomic theory? In one book L is given as an energy shell and in one book L is represented as the orbital angular momentum of an electron. So I want to say that the energy shell can also be represented as 1, 2, 3 but orbital angular momentum is always represented as L so which one is right?

> **Answer:** Atomic physics notation can be confusing. In atomic physics, capital letters, starting with K, are often used to denote shells containing electrons with the same principal quantum number n: $n = 1$ is the K shell, $n = 2$ is the L shell, $n = 3$ is the M shell, etc. The magnitude of the total orbital angular momentum of a quantum state is almost always denoted by L. The value of L is determined by the orbital angular momentum quantum number ℓ: $L = \hbar\sqrt{}[\ell(\ell + 1)]$ where $\ell = 0, 1, 2, 3\ldots$ (This is easy to miss: unitalicized L denotes the $n = 2$ shell, *italicized* L denotes the orbital angular momentum of an electron in a quantum state specified by the orbital angular momentum quantum number ℓ.) Also of importance is the projection (component) of the angular momentum on to a z-axis, $L_z = m_\ell\hbar$, where $m_\ell = -\ell, -\ell + 1, -\ell + 2 \ldots 0 \ldots \ell - 2, \ell - 1, \ell$. But it still gets a little crazier! The common notation is to specify n and ℓ by a number (n) followed by a letter (s for $\ell = 0$, p for $\ell = 1$, d for $\ell = 2$ and f for $\ell = 3$). This peculiar labeling of angular momentum quantum numbers is a historical artifact where the words sharp, principal, diffuse and fine were used to describe spectral lines. After f, the letters proceed alphabetically—g for $\ell = 4$, h for $\ell = 5$, etc. So, for example, an orbital labeled 3p refers to an electron with $n = 3$, $\ell = 1$ and the ground state of a hydrogen atom is 1s.

Note that although the Bohr model gets the energies right, its principal assumption, that $L = n\hbar$ is completely wrong. The ground state ($n = 1$) has zero angular momentum, the first excited state ($n = 2$) is really two states with the same energies (states with the same energies are called *degenerate*) and angular momentum is either $L = 0$ of $L = \hbar\sqrt{2}$, and so on. In this regard, it is interesting to note that the Planck model for a black body was also slightly wrong; instead of $E = nhf$, $n = 1, 2, 3, \ldots$, the correct expression is $E = (n + \frac{1}{2})hf$, $n = 0, 1, 2, \ldots$.

Now, it would seem that all we need to do to obtain a model of any other atom is to increase the charge of the nucleus by e and add an electron, continuing to do this as we move higher and higher into the periodic table. It would be a little more difficult because we would have to include the repulsive forces between electrons, each of which would also see a larger attractive force toward the nucleus. But, supposing the electron–electron force is a minor perturbation, we would just have Z electrons in the 1s shell. After all, there is no reason that every electron would not want to be in the lowest level, is there? It turns out that there is.

3.6.2 Spin and the Pauli exclusion principle

There is still one thing we have not thought of. In the answer to the question below, I allude to the Earth having both orbital angular momentum (the motion around the Sun) and intrinsic angular momentum (the motion about its own axis). What if such a thing were found for elementary particles?

> **Question:** My physics lecturer tried his best to answer my question on what exactly the spin of an electron is. But I couldn't understand. Besides he himself looked quite confused. So could you kindly help me out? What is generally a spin of an electron? And how would I explain it to a layman?

> **Answer:** Take the Earth as an analogy. Earth has two kinds of angular momentum—it revolves around the Sun and it rotates on its axis. Angular momentum refers to rotational motion of something. Its motion around the Sun is called *orbital angular momentum* and its spinning on its axis is called *intrinsic angular momentum*. Now think about an electron in an orbit around a nucleus in an atom. It clearly has orbital angular momentum because of its orbit. But it also has intrinsic angular momentum, that is, it behaves in many ways as if it were spinning on its axis; this is usually referred to simply as *spin*. This is a fine analogy, but it should be emphasized that spin is not really such a simple classical concept. For example, if you were able to exert a big enough torque on the Earth you could stop its spinning or make it spin faster. The spin of an electron is a constant property and cannot be changed. You can change the direction (clockwise or counterclockwise) but not the magnitude of spin. Also, if you try to model the electron as a little spinning sphere to find out how fast it is spinning, you obtain absurd, unphysical answers. So, you can only push the electron/Earth analogy so far. Finally, I should note that in relativistic quantum mechanics (the Dirac equation) the existence of electron spin is predicted to be exactly what it is measured to be.

So, if an elementary particle like an electron were to have an intrinsic angular momentum, we would expect, on the basis of our experience with the hydrogen atom orbital angular momentum, that it would be characterized by some quantum number s (called spin) such that the intrinsic angular momentum S would have a value $S = \hbar\sqrt{[s(s+1)]}$. We would also expect s to be some integer, since that seems to be the way that angular momentum is quantized, again based on our understanding of the hydrogen atom.

The first experiment to observe spin was performed by the German physicists Otto Stern and Walther Gerlach in 1922. They were in fact trying to provide experimental support for the Bohr model of atoms. Looking at the chronology of developments in quantum physics, appendix H, you will see that 1922 predates both the idea of intrinsic spin (1924–5) and the Schrödinger equation (1925); therefore they could not have been looking for spin. The idea was that, if electrons were in orbits, those orbits would look like tiny bar magnets and could therefore be detected by their interaction with magnetic fields. (A current loop has a magnetic field which behaves just like a tiny bar magnet.) The details of the Stern–Gerlach experiment are presented in appendix G. The experiment found that a beam of silver atoms, passed through a nonuniform magnetic field, were deflected half up and half down. This was an indication of some kind of quantization, but what it was not understood until the idea of spin was proposed.

Let us now interpret the experiment in hindsight. If the spin quantum number s were an integer, then the beam should have been split into an odd number of deflections because what the experiment was observing was the m quantization, orientation of the spin. For example, if $s = 1$, $m_s = -1, 0, 1$, three deflections, one up, one down and one straight through. If there were only two deflections, then there was no $m = 0$, so the conclusion had to be that $s = \frac{1}{2}$, $m_s = \pm\frac{1}{2}$ and $S = \hbar\sqrt{(3/4)}$. Now, we can write the complete quantum picture of the hydrogen atom. An electron may have four quantum numbers, n, ℓ, m_ℓ and m_s. Before we knew about spin, we concluded that each energy level specified by n would have n^2 different possible states; since for each of these we can now have $m_s = \pm\frac{1}{2}$ the correct number of degenerate states for each principal quantum number is $2n^2$.

The following Q&A provides a review of what we have been talking about in this book as well as a nice introduction to the important Pauli exclusion principle.

> **Question:** I keep hearing about particles having intrinsic properties known as ‘spin’. I looked it up, and all I got from it is that it divides particles into different groups (bosons and fermions) and that it is closely related to angular momentum. I think I understand what angular momentum is, but I don’t see how it is measured in numbers. Could you explain that for me? Also, what is spin exactly? I understand it is also related to some sort of rotation symmetry. How’s a spin 1 particle different from a spin 2 particle? Why are particles with half integral spin placed in a separate group?
>
> **Answer:** Let’s think of a classical particle—think of a bowling ball. What intrinsic properties can it have? Well, it has mass, so it also has rest-mass energy. If this particle retains its identity, then its mass is something which you cannot change. If it is moving it has linear momentum and kinetic energy. These can be changed by changing the velocity of the particle; in fact we could put the bowling ball at rest and it would have no kinetic energy or linear momentum. It might be spinning about its center of mass, so it would have angular momentum and rotational kinetic energy. These can be changed by changing the angular velocity of the particle; in fact we could stop the ball’s spinning and it would have no rotational kinetic energy or angular momentum. These are the main things; we could subject it to some force field (like gravity) and talk about its potential energy as an intrinsic property, but it is easier to just restrict ourselves to talk about bowling balls in the middle of empty space.
>
> Now let’s think about a particle like an electron or an atom or a proton—something in the microscopic realm. What makes it any different from a bowling ball apart from the fact that it has less mass? Well, if you were a 19th century scientist, you would see no reason why there would be anything at all different. But when detailed measurements on such small systems started to be made, it was found that expectations of their properties based on ‘old fashioned’ classical ideas turned out not to work. Quantum mechanics was born to explain the strange new phenomena. A particle still has a mass and a rest-mass energy. A particle still has linear momentum and kinetic energy, but they behave strangely. If it is a free particle, its momentum cannot be precisely known unless we are totally ignorant of its position and its kinetic energy cannot be precisely known unless we have forever to measure it; these are the results of the famous Heisenberg uncertainty principle. What this means, essentially, is that a particle may not be put perfectly at rest somewhere. Finally we come to what you are interested in: can a tiny particle, say an electron, be spinning? The answer is yes, but only with the discrete amount of angular momentum which nature gives it. For particles in quantum

mechanics, the spin angular momentum has a magnitude of $\hbar\sqrt{[s(s+1)]}$ where $\hbar$ is a physical constant called the rationalized Planck's constant, and where s is the spin quantum number for the particle or state of the particle. s is called the spin of the particle and may only take values of 0, 1/2, 1, 3/2,..., $n/2$ where n is an integer. And it is *unchangeable*: an electron (which has $s = 1/2$), for example, may not be stopped from spinning, it always has a spin angular momentum of $\hbar\sqrt{(3/4)}$. This is just the way that nature is! Why do particles have spin? It turns out to be a relativistic thing. If you do quantum mechanics relativistically, spin just comes out of the theory.

Finally, what can you tell about a particle by knowing its spin? It turns out that spin determines the 'statistics' of the particle. What 'statistics' means here is that nature seems to have divided its particles into two types:

1. Particles which, if in some quantum system with other identical particles (e.g. a bunch of electrons in an atom), are forbidden to be in exactly the same state as any other particle in the system. These particles are called *fermions.* (Incidentally, 'state' means all the things which are needed to characterize a particle—energy, orbital angular momentum, spin angular momentum, magnetic quantum numbers, etc.)
2. Particles which are not subject to this restriction. These are called *bosons.* The restriction is called the Pauli exclusion principle.

What determines whether something is a fermion or a boson is its spin. If the spin is 0, 1, 2,... it is a boson; if it is 1/2, 3/2, 5/2,... it is a fermion. It is a lucky thing that electrons are not bosons since all atoms would have essentially the same chemistry as hydrogen and there would be no you.

3.6.3 Atoms beyond hydrogen

The key to understanding why atoms with different numbers of electrons (specified by Z, atomic number) have different properties can now be understood. The key is that electrons, with $s = \frac{1}{2}$, are fermions, so as each energy shell is filled, added electrons go into higher and higher shells because the Pauli exclusion principle forbids there being more than one electron in an atom with a particular set of quantum numbers. Keep in mind that it is an approximation to just think of adding electrons to the energy levels of hydrogen because to do so ignores all the complications which come from solving the Schrödinger equation for a many-electron system—the forces between each electron and the $Z - 1$ others and magnetic interactions between the electron spin and the magnetic field caused by the orbiting electrons (called the spin–orbit force).

Table 3.1 shows a few light atoms. I have listed only the n and ℓ quantum numbers participating for each element, but be sure to keep in mind that for each value of ℓ there are $2\ell + 1$ possible electrons (the number of m_ℓ quantum numbers) and for every possibility for n, ℓ and m_ℓ there are two values of m_s. The configuration for each atom is easy to read in this table if you remember that s means $\ell = 0$ and p means $\ell = 1$; for example, neon (Ne) has a configuration $1s^2 2s^2 2p^6$ which means

Table 3.1. Electrons in selected light atoms.

Atom	Z	n	ℓ	Configuration	Category
hydrogen (H)	1	1	0	$1s^1$	—
helium (He)	2	1	0	$1s^2$	Noble gas
lithium (Li)	3	1, 2	0, 1	$1s^22s^1$	Alkali metal
beryllium (Be)	4	1, 2	0, 1	$1s^22s^2$	Alkali earth
neon (Ne)	10	1, 2	0, 1	$1s^22s^22p^6$	Noble gas
sodium (Na)	11	1, 2, 3	0, 1	$1s^22s^22p^63s^1$	Alkali metal
magnesium (Mg)	12	1, 2, 3	0, 1	$1s^22s^22p^63s^2$	Alkali earth
argon (Ar)	18	1, 2, 3	0, 1	$1s^22s^22p^63s^23p^6$	Noble gas
potassium (K)	19	1, 2, 3, 4	0, 1	$1s^22s^22p^63s^23p^64s^1$	Alkali metal
calcium (Ca)	20	1, 2, 3, 4	0, 1	$1s^22s^22p^63s^23p^64s^2$	Alkali earth

there are two electrons in the $n = 1$, $\ell = 0$ orbit, two electrons in the $n = 2$, $\ell = 0$ orbit and six electrons in the $n = 2$, $\ell = 1$ orbit. I have chosen a few of the lighter atoms to illustrate how what you now know about atomic structure can qualitatively explain the periodic table which classifies elements by their chemical properties.

Lithium (Li), sodium (Na) and potassium (K) all are highly reactive metals. If you put a chunk of any of these in water (don't do it!) they react violently creating hydrogen gas and a hydroxide of the metal. You can easily see that the last electron to be added was an s ($\ell = 0$) electron. Hydrogen also fits this prescription, but is generally not referred to as an alkali metal.

If you add one more electron to an alkali metal, the s orbital is now filled, ns^2, and that characterizes what is called an alkali earth, beryllium (Be), magnesium (Mg) and calcium (Ca) in my examples. These all have similar chemical properties.

Finally, just before the alkali metals, are the noble gasses, helium (He), neon (Ne) and argon (Ar). These elements are totally nonreactive, they do not form molecules with other elements. Since (as we will see later) the s orbital is always the lowest for a given principle quantum number n, we can say that the $n - 1$ shell has just filled for noble gasses; the nomenclature is that all major shells are full in these elements.

The chemical properties of atoms, as you can surmise, are determined by the last few electrons added and, for the most part, the inner electrons remain as an inert core. Those outer electrons are called valence electrons by chemists.

3.6.4 More details of multi-electron atoms

Before ending this chapter, we should learn a bit more about the consequences to having many electrons all interacting with each other. The following question about lithium, a fairly simple atom, illustrates how the simple structure of hydrogen is changed when more atoms are added.

> **Question:** We did a lab where we put lithium in a flame and saw it emitted red light. We were told this happens because an electron gets so much energy it jumps from one electron shell to the next. Then when it falls back to the lower energy

level, it gives off photons of light. We are having trouble understanding what is happening to the electrons. Lithium has two electrons in the first energy level and one in the second. Does an electron in the first shell jump to the second? Or does an electron in the second jump out to the third? And if it is an element with three orbitals, does an electron jump from the first to the second, and if it does, does an electron also move from the second to the third? How does the movement of electrons from one shell to the next affect the other electrons in that shell?
Answer: Figure 3.10 shows the energy-level diagram of lithium. The thing to understand is that the two inner electrons (in the 1s shell) are essentially inert for your experiment, only the outer electron is excited. So the outer electron (in the 2s shell) looks in and sees the nucleus, charge +3, shielded by two electrons, charge −2, for a net charge of +1. In other words, the lithium spectrum should look a lot like the hydrogen spectrum because the active part of the lithium atom looks pretty much like a hydrogen atom. Note, for comparison, the energy levels for hydrogen shown on the figure. The red light, with a wavelength of about 670 nm, results from the transition from the first excited state (2p) to the ground state (2s).

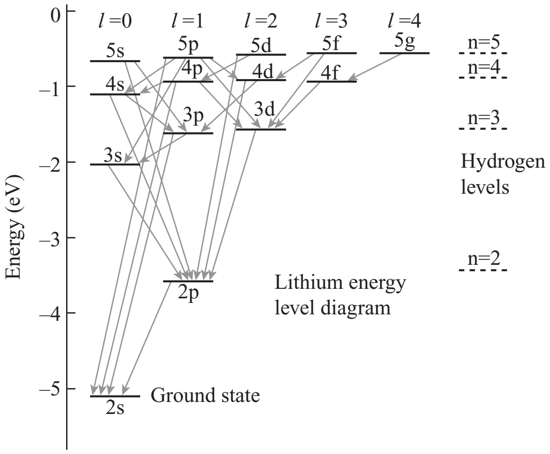

Figure 3.10. Energy-level diagram for lithium.

Figure 3.10 is called an energy-level diagram and, since this is the first time we have seen one, an explanation is in order. The vertical axis is the energy of the atom. This energy is always negative which indicates that the atom is bound. For example, the ground state of lithium has an $n = 2$, $\ell = 0$ orbit; figure 3.10 labels this level 2s and the energy is about −5.1 eV. If you add 5.1 eV to this atom, this electron will be removed from the atom, so, you see, the energy being negative makes sense, $-5.1 + 5.1 = 0$. Energy levels are plotted as horizontal lines and generally labeled by their quantum numbers. Often, as in figure 3.10, the horizontal axis is used to characterize some other aspect of the level; in figure 3.10 the horizontal displacement is used to separate the different ℓ values. Energy diagrams are often also used to

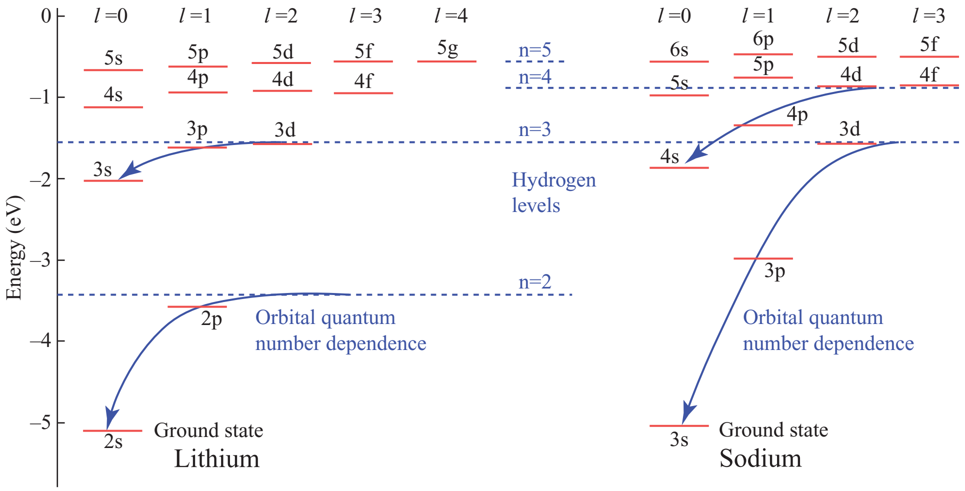

Figure 3.11. Energy-level spectra of Li and Na.

show the transitions which result in the spectrum of the element; arrows are drawn between the participating levels. Figure 3.10 shows the main transitions in the Li atom and the 2p → 2s transition is the red light referred to in the question.

One of the most striking things about the spectrum of lithium is that, unlike hydrogen, all the ℓ values for a given n do not have the same energies (are no longer degenerate). For each value of n, the lowest-energy orbit is the s ($\ell = 0$), the next is the p ($\ell = 1$), and so on until ($\ell = n - 1$) is reached. This is shown in figure 3.11 for both lithium and sodium. Here the blue arrows are not transitions but drawn to show the trends: the higher values of ℓ are close to the corresponding hydrogen energy but lower ℓ are pushed down in energy. As more electrons are added this effect becomes larger as can be seen by the comparison of the two energy-level diagrams. Apparently an electron would much prefer to be in an s orbital than a p orbital.

Finally, if you examined table 3.1 carefully, you would have noted a puzzling thing. Why is the ground-state configuration of potassium $1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$? The $n = 3$ shell should include $\ell = 0$, 1, 2. So, where are the 3d electrons? Why is the ground-state configuration not $1s^2 2s^2 2p^6 3s^2 3p^6 3d^1$? Why is potassium an alkali metal? The answer is clear in figure 3.11. Because of the orbital quantum number dependence of the orbital energies, the 4s energy is lower than the 3d energy.

Chapter 4

Nuclear physics

4.1 Introduction

In chapter 3 the nuclear model of atoms was introduced. You will recall that Rutherford's explanation for the α-particle scattering data of Geiger and Marsden was that nearly all the mass of the atom was concentrated in a volume whose radius was on the order of several femtometers ($1\ \mathrm{fm} = 10^{-15}\ \mathrm{m}$). This chapter will discuss the properties of the nuclei of atoms, in particular

- the nuclear force,
- radioactivity,
- their composition,
- their sizes and masses,
- fission, and
- fusion.

The experimental study of the properties of nuclei is much more challenging than for the atom; because it is so small and positively charged, it requires very high-energy particles to probe it. Even in the Geiger and Marsden experiments, α-particles on the order of a few million electron-volts were unable to get close enough to 'touch' the nucleus.

Again, this topic would be easier to understand if *From Newton to Einstein* were read first because special relativity is fully developed there. You can understand most of what is necessary just by knowing that $E = mc^2$ and that you change the energy of something if you do work on it (or it does work for you). Appendix B will give you all you need to know about energy.

4.2 Discovery of the neutron

Although radioactivity was discovered long before the nucleus itself (see appendix H), it is best to understand the components of the nucleus before discussing radioactivity.

doi:10.1088/978-1-6270-5939-8ch4

The net charge of an atom, unless electrons are removed from it, is zero, so the charge of the nucleus must be $+Ze$ where Z is the atomic number and $e = 1.6 \times 10^{-19}$ C is the electron charge. We know that the nucleus of a (normal) hydrogen atom is a single particle called a proton. We also know that the α-particle, such as Rutherford used in his scattering experiments, is the nucleus of a helium atom and so it has a charge of $2e$; however, it has a mass roughly four times that of a proton. So the question naturally arises, what is a nucleus of an element with $Z > 1$ made of?

Roughly speaking, the mass of an atom is found to be close to an integer multiple of the mass of a hydrogen atom and that integer is called the atomic weight and denoted by A. The first model proposed for the nucleus was that it consisted of A protons and $A–Z$ electrons (called nuclear electrons). There were numerous arguments against this model, though. If you estimate the kinetic energy which an electron would have if confined to so small a volume, it turns out to be tens of MeV; representative calculations illustrating this are given in appendix I.1. For comparison, electron energies in the atom are a few eV (shown in appendix I.2). But through studies of the radioactivity of nuclei, which we will discuss later in this chapter, it was known that nuclear energies are on the order of a few MeV or less making nuclear electrons very unlikely. Also, once spin was discovered, it was clear that the spins of nuclei were incompatible with the nuclear electron model. Finally, the uncertainty principle for electrons confined to the nucleus would imply that $\Delta p \sim \hbar/\Delta x$. This would imply an uncertainty in the energy of the electron of about 20 MeV; however, electromagnetic radiation from nuclei (γ-rays) were known to be sharp lines like atomic spectra.

In 1931–2 several experiments involving bombarding very light atoms with energetic α-particles resulted in a very penetrating radiation. It was at first assumed that this radiation was γ-rays (high-energy photons) which were known to be able to penetrate far through materials. A particularly important experiment was performed by Irène Joliot-Curie and Frédéric Joliot in 1932; the 'mystery radiation' was directed on sheets of paraffin and resulted in very energetic protons, on the order of 5 MeV kinetic energy, being ejected in the same direction as the radiation. James Chadwick, a colleague of Rutherford's at Cambridge, realized that the radiation could not be photons because their energy would have to have been greater than 50 MeV; this is shown in appendix I.3. As stated above, this energy is completely incompatible with what was known about nuclear energies.

Chadwick was subsequently able to demonstrate that the mystery radiation must be composed of uncharged particles with very nearly the same mass as the proton; these new particles were called neutrons. You can understand this by visualizing a game of pool: if the cue ball strikes a ball at rest head-on, the cue ball remains at rest after the collision and the struck ball exits with the same velocity the cue ball originally had. We will come back to this visualization later when we discuss the role of neutrons in nuclear reactors.

The model of the nucleus was now understood to be Z protons and $N = A–Z$ neutrons, collectively called nucleons. Now that we have a pretty good idea of the constituents of the nucleus, neutrons and protons, we can proceed with talking about nuclear properties.

4.3 The nuclear force (the strong interaction)

Question: If protons repel protons, how is it possible for them to be in the nucleus?
Answer: This is how we know that there is another force present besides the electrostatic repulsion. It is called the nuclear force or strong interaction. This force is very short-ranged. That is, if the protons are not very close to each other, this force will be very small and the repulsion will win out; but if they are very close, the nuclear force wins out and a nucleus may be held together. Neutrons also feel this force which is why neutrons are in nuclei.

This question is the obvious one to ask—what keeps a nucleus together? There must be some other force beyond the repulsive electrical force to hold it together. To get a feeling for the energy (E) required to bring two protons to a separation $r = 1$ fm (10^{-15} m), use the equation $E = e^2/(4\pi\varepsilon_0 r)$ where $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 (N · m^2)$^{-1}$ (see appendix A) and $e = 1.6 \times 10^{-19}$ C is the proton charge. Doing the calculation, $E = 2.3 \times 10^{-13}$ J = 1.4 MeV. And, to get a feeling for the force necessary to hold the two protons separated by 1 fm, $F = e^2/(4\pi\varepsilon_0 r^2) = 230$ N. This may not sound like much of a force, but the acceleration of a 1.7×10^{-27} kg proton would be $230/1.7 \times 10^{-27} = 1.35 \times 10^{29}$ m s^{-2}! 230 N is huge to a proton. That is why they call the nuclear force which balances this electrical force *the strong interaction*.

Question: I have been told that the strong force becomes repulsive at small distances. Is this the case and can you explain why or why not?
Answer: It is certainly true. I cannot explain why since that is not really the goal of physics; we don't, for example, ask for an explanation of why the electron is negative, it just is. The veracity of the repulsive short-range force is easy to understand. If it were not so, the nucleus (held together by the strong interaction) would collapse. This is called saturation of nuclear forces.

So, the force is strongly attractive to overcome the repulsive electrical force and it must also be very short-ranged or else nuclei would be much larger. But as the Q&A above indicates, there must be something about it which keeps the whole nucleus from collapsing into a point. At very short ranges, around 1 fm, this strongly attractive force abruptly becomes strongly repulsive. Now that we have an idea of what nuclei are made of and what the nature of the force holding nuclei together is, we can begin exploring the properties of nuclei.

The notation commonly used to specify the nucleus you are talking about is ${}^{A}_{Z}X_N$ where X is the symbol for the element, A the atomic weight, Z the atomic number and N the neutron number. Usually N is not specified since it is implicitly specified because $N = A–Z$; in fact, since the element (X) implicitly tells you the atomic number, Z is often not used and the notation is simply ${}^{A}X$. Nuclei which have the same number of protons (Z) but different numbers of neutrons (N) are called isotopes. Most elements have more than one isotope. For example, copper (Cu), has two stable isotopes ${}^{63}_{29}\mathrm{Cu}_{34}$ and ${}^{65}_{29}\mathrm{Cu}_{36}$ (or just ${}^{63}\mathrm{Cu}$ and ${}^{65}\mathrm{Cu}$). Stable means that they live forever unless you do something to them. Copper also has numerous known unstable isotopes, from ^{52}Cu to ^{80}Cu; these isotopes, if created, will transmute into other nuclei by various mechanisms which are called radioactivity. Radioactivity will be discussed in section 4.6.

4.4 Nuclear masses and binding energies

Suppose that you measure the rest-mass energy of some particular isotope; let's take ^{65}Cu as an example where $M_{\mathrm{Cu}}c^2 = 58618.54$ MeV. Hydrogen has $M_{\mathrm{H}}c^2 = 938.78$ MeV and the neutron has $M_{\mathrm{n}}c^2 = 939.57$ MeV. So, a ^{65}Cu atom has 29 hydrogen atoms and 34 neutrons, so the total rest-mass energy of all the constituents of ^{65}Cu is $29 \times 938.78 + 34 \times 939.57 = 59170$ MeV. Here is a remarkable thing, a nucleus weighs significantly less than the sum of the weights of its components! $B = 59170 - 58618.54 = 551.46$ MeV where B is called the binding energy. Later, the binding energy per nucleon, B/A, will be useful, so for ^{65}Cu, $B/A = 551.46/65 = 8.48$ MeV/nucleon.

So, how can this be, that the whole is less than the sum of its parts? It is very simple, actually. Do you have to do any work to remove a nucleon from a nucleus? Of course, you have to pull it out of its attachment to its compatriots. So if you remove one nucleon from a bound state, you add energy to the whole system. And if the one you pulled out is now very far away and at rest, what happened to that energy? If everything is at rest, there is no kinetic energy. It shows up in the change in rest mass, something on the order of 8 MeV to remove a nucleon from copper. That is a lot of energy for this tiny system—the mass of the ^{65}Cu is more than half the mass of a nucleon lighter than the summed mass of its components. This is completely contrary to the experience from chemistry where, early in the development of the science, the 'law of conservation of mass' was gospel—the total mass going into a chemical reaction was always equal to the mass going out.

> **Question:** My question is about the law of conservation of mass/matter. While the Universe is expanding, does this law apply? Why or why not?
> **Answer:** There is no such thing as conservation of mass/matter; in chemistry, it was one of the keystones: combine 16 g of oxygen with 2 g of hydrogen and obtain 18 g of water. However it turns out that this is not quite true because of $E = mc^2$; some energy is released when you burn hydrogen and this consumes a little mass. However, the law is very close to true for chemistry because the energy released is tiny compared to the energy of all the mass. But it is not true for nuclear fusion which is what fuels stars like the Sun and these stars become measurably lighter as they age. And it is not true for nuclear fission which is what fuels nuclear power plants. What is true is conservation of energy (including mass energy) for an isolated system.

Where does energy come from when you do chemistry, burn fossil fuels for example? It comes from mass. If you could make a careful-enough measurement of burning carbon to obtain carbon dioxide, you would find that there was a tiny amount of mass missing. But, recall from chapter 3 that atomic energies are on the order of eV whereas we have learned in this chapter that nuclear energies are on the order of MeV. So mass changes in chemistry are a million times smaller than mass changes in nuclear reactions, small enough to never be noticed. This is probably a lucky thing since so much of the development of chemistry depended on conservation of mass. Still, chemistry is a pretty inefficient way to obtain energy.

Back in the 19th century, many physicists tried to estimate how long the Sun would last and how long it had been burning based on the assumption that some fuel was 'burning', i.e., chemistry was fueling the Sun. These estimates came up with about 50 000 years of total lifetime. However there was already a lot of evidence in the 19th century from geology that the Earth was at least hundreds of millions of years old. In fact, we know today that the solar system is about 4.6 billion years old and the Sun has an expected lifetime of about 10 billion years. Nuclear energy, as we shall see in section 4.8, is the power house of the Sun.

4.5 Nuclear systematics

Before discussing radioactivity, it will be useful to discuss some systematic properties of nuclei across the whole range of known elements.

Which isotopes of which element are stable and which are unstable can be pretty well illustrated by what is called a *chart of the nuclides*, shown in figure 4.1. This is a grid with N on one axis and Z on the other. We will be referring to this chart many times in this chapter. The most important thing to note now is the stable nuclei, shown in black. There is a quite narrow band up to $Z = 82$, lead. The band of stable isotopes begins along $N = Z$ but after $Z = 20$, neon, there are no stable nuclei with equal numbers of protons and neutrons. The vast majority of nuclei have $N > Z$, the excess of neutrons becoming as large as 50% for the heaviest nuclei, for example lead ${}^{208}_{82}\mathrm{Pb}_{126}$.

All other nuclei are unstable and the vast majority decay by α- and β-decay which will be discussed in sections 4.6.3–4.6.5. Nearly all the radioactive nuclei shown do not exist in nature, they have been made in reactors and accelerators. Very rare decays are proton- or neutron-decay; only if there is an extreme excess of neutrons or protons will there be emission of a nucleon.

How can you tell whether any particular nucleus will be radioactive or not? Surprisingly, the main constraint is whether it is energetically possible or not. For example, suppose that you start with some heavy nucleus, say polonium-210, ${}^{210}_{84}\mathrm{Po}_{126}$ and you wonder if it could decay by emitting a helium-4 nucleus ${}^{4}_{2}\mathrm{He}_{2}$ (an α-particle). The decay product, or daughter, would be lead-206 ${}^{206}_{82}\mathrm{Pb}_{124}$. Calculate the total mass energy before and after the decay:

$$M_{\text{before}}c^2 = M_{\text{Po}}c^2 = 195594.85\,\text{MeV},$$
$$M_{\text{after}}c^2 = M_{\text{Pb}}c^2 + M_{\text{He}}c^2 = 191861.09 + 3728.34 = 195589.43\,\text{MeV}.$$

The difference of the initial and final masses is called the Q-value.

$$M_{\text{before}}c^2 - M_{\text{after}}c^2 = 5.42\ \text{MeV} \equiv Q$$

The value of Q tells you whether the decay can happen energetically. In this example there is an excess of 5.42 MeV after the decay which will show up as the kinetic energy of the emitted α-particle. It was energetic α-particles like these which Geiger, Marsden and Rutherford used in their experiments which led to the discovery of the nucleus. In general, if Q is positive, the reaction is energetically allowed and will happen if there is no other constraint. If Q is negative, this means that you would have to add energy to make the decay occur. Of course there are other constraints which could prevent a decay from happening; these are called selection rules which we will not go into in this book. You also cannot violate conservation laws; for example you could not have a decay which had more electric charge after the decay than before. Since nearly all known isotopes are unstable, this means that nature can usually find a way of changing into a system which has less mass than you started with; it does not end until you end up on a stable nucleus.

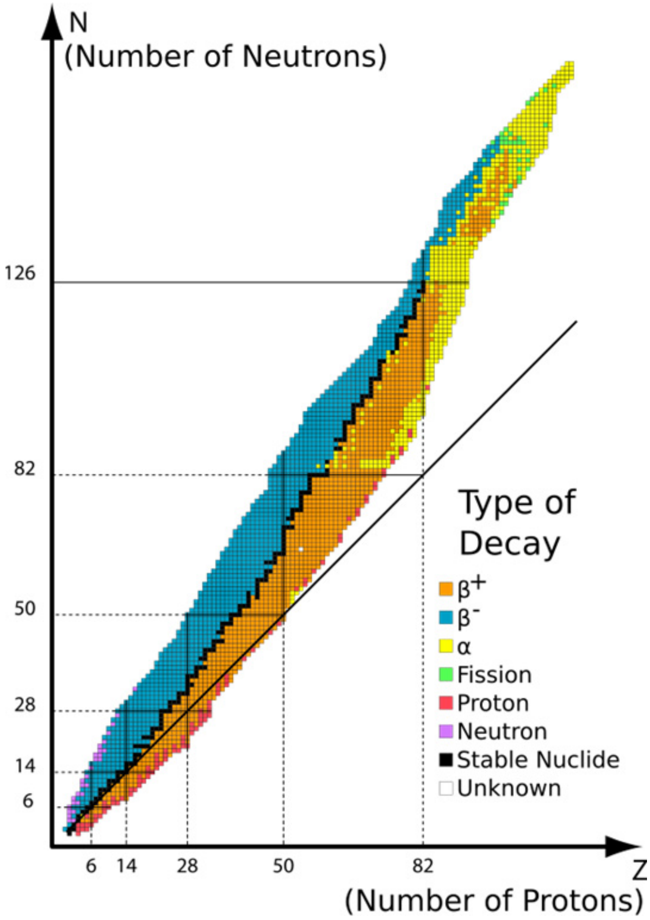

Figure 4.1. Chart of the nuclides.

4.6 Radioactivity

4.6.1 Introduction

As discussed earlier in this chapter, most nuclei are unstable, they disintegrate in some way and transmute to other nuclei. The purpose of this section is to understand the ways in which unstable nuclei decay. This decay is called radioactivity. Radioactivity was discovered by Henri Becquerel in 1896. He was studying radiations from a uranium compound and his apparatus had a magnetic field to deflect charged radiation. There were three main types, with an electric charge zero, a negative charge and a positive charge. Subsequently, radioactivity was studied by Marie and Pierre Curie who gave radioactivity its name and discovered radium and polonium and by Ernest Rutherford who named the three types alpha, beta and gamma for the positively, negatively and zero-charged components, respectively. They were named in order of their penetrating ability—alpha the lowest and gamma the highest. These three classes of radiation will be discussed in this chapter.

4.6.2 Half-life

Half-life is often talked about when radioactivity is discussed. It is simply the time it takes for the population of radioactive nuclei to reduce by half. This number is independent of how many nuclei you start with; after one half-life you have $\frac{1}{2}$ the original, after two half-lives $\frac{1}{4}$ the original, after three half-lives $\frac{1}{8}$ the original, after n half-lives, $\frac{1}{2^n}$ the original. Examples are shown in figure 4.2 where decay curves are shown for a half-lives of about 0.7 s (black) and 0.35 s (red). The basic idea is that it is equally probable that any nucleus will decay at any instant and therefore that the rate of decay is proportional to the number of radioactive nuclei—twice the number of nuclei will mean twice the rate of decay. It is important to understand

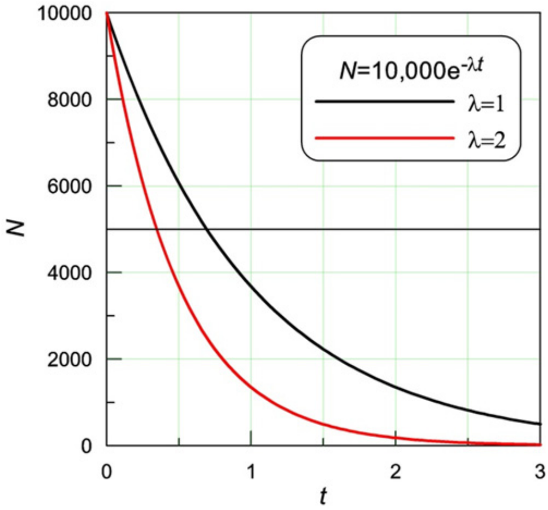

Figure 4.2. Exponential decay.

that half-life is a statistical concept and requires a large sample; it would be pointless to try to assign a half-life to a sample of ten nuclei. This is rarely an issue for radioactive nuclei which usually are present in large numbers. The following Q&A addresses the mathematical details of the decay.

> **Question:** I have a question on the law of decay. I never heard of it before and I am quite confused. It's about a radioactive material that is known to decay at a rate proportional to the amount present. What does the mass of the material initially have to do with the time taken for the material to lose some percentage of its mass?
> **Answer:** This is very fundamental and can apply either to decay or growth. The idea is that the rate of decay or growth is proportional to the population. Does it make sense? Suppose we have 10 000 radioactive nuclei and we measure that they are decaying at a rate of 50 s^{-1}; it is then perfectly reasonable to assume that if we had 20 000 instead, we would see 100 s^{-1} decaying. It simply reflects the situation that it is equally probable for any of the population to decay in the next 5 s, for example. It also applies to many growth problems. For example, if you have 10 000 bacteria and they are growing at the rate of 50 s^{-1}, the rate of growth of 20 000 of them would be 100 s^{-1}. The general way to write this down (I assume you know some calculus?) is $\mathrm{d}N/\mathrm{d}t = \pm\lambda N$ where N is the number at any time t, the + sign is for growth, the – sign is for decay and λ is called the decay constant, the proportionality constant. You now have to solve this equation for N. It is a basic differential equation, easily shown to have a solution $N = N_0\mathrm{e}^{\pm\lambda t}$ where N_0 is the number at the time $t = 0$. Figure 4.2 shows two decay curves with $N_0 = 10\,000$, the black curve with $\lambda = 1\ \mathrm{s}^{-1}$ and the red curve with $\lambda = 2\ \mathrm{s}^{-1}$. λ may also be related to the half-life $T_{½}$, the time it takes the initial population to drop to $N_0/2$, by $T_{½} = 0.692/\lambda$; note that the black curve is at 5000 at about $t = 0.7$.

4.6.3 α-radiation

α-radiation is the emission by mainly very heavy nuclei of ^{4}He nuclei (2 protons, 2 neutrons). (See figure 4.1, the yellow grid points.) As noted in the following answer, the α-particle is very tightly bound making it likely that one will form inside a heavy nucleus. It is not very penetrating, being easily stopped by a few inches of air or a sheet of paper. The heavy, doubly charged ion quickly loses its energy to whatever atoms it encounters.

> **Question:** What about the structure of α-particles makes them such a common form of radiation? Why aren't, for instance, lithium nuclei the common form of ionizing radiation?
> **Answer:** α-radiation is something which happens mostly for very heavy nuclei. Think of all the protons and neutrons buzzing around inside the nucleus, all interacting with each other. It turns out that the α-particle (^{4}He, 2 protons and 2 neutrons) is the most tightly bound of the light nuclei. So the alpha is tightly bound and has few particles and the result is that it has a relatively high

> probability of spontaneously forming inside the nucleus. The energetics are such that the total energy of the original nucleus ${}^{A}_{Z}X_{N}$ is greater than the energy of an α-particle plus the daughter nucleus ${}^{A-4}_{Z-2}W_{N-2}$ and so α-particle decay will happen if a mechanism for decay can be found. It gets a little complicated here because the α-particle, if very close to the daughter nucleus (which being inside certainly is) is strongly bound; but if it could 'figure out' a way to reach a distance a little outside the surface of the daughter nucleus, the electrical repulsion would be bigger than the nuclear attraction and it would shoot out. The mechanism for getting away is called quantum tunneling and there is also a probability that an α-particle will tunnel out and escape. Heavier nuclei (like Li) have a lower probability of formation and a lower probability of tunneling out if they do form. Roughly speaking, the probability of α-decay is the product of the probabilities of formation and tunneling. You could also think of α-decay as a very asymmetric fission of a heavy nucleus.

The 'little complicated' part referred to in the answer above, tunneling, is very interesting and may be worth a brief explanation here. If an α-particle forms inside the nucleus, it will approximately see that it can move about inside the nucleus but cannot escape because the nuclear force is holding it from getting out. But if it were just a little way outside the surface of the nucleus, just outside the range of the nuclear force, it would see the Coulomb repulsion pushing it away. Figure 4.3 shows this schematically, the red line representing the force (it is actually the potential energy, for readers conversant with that). What is shown is the wave function of the α-particle. When inside a sphere of radius R_0, it sees a wall it cannot penetrate. But wait, that is the classical view; in quantum mechanics, the α-particle could exist in

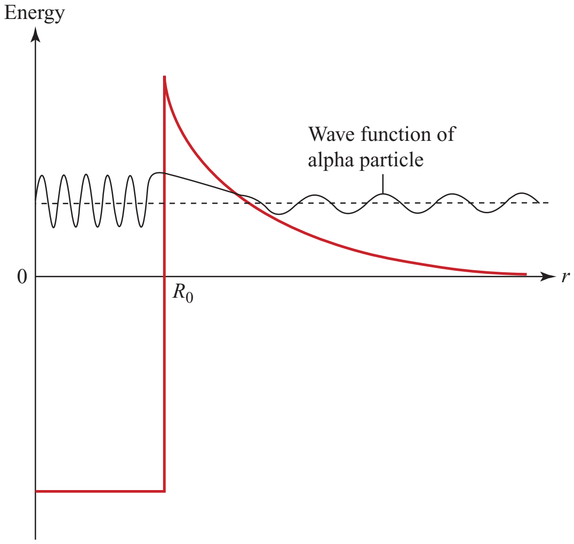

Figure 4.3. α-particle tunneling.

the region between the inside and outside of the red line and there would be a small probability of its existing outside. The 'thicker' this wall, the harder for an α-particle to tunnel out, so the longer the half-life. Half-lives of alpha emitters are typically days to hundreds of thousands of years. The explanation for α-decay was devised by George Gamow (1904–68) in 1928.

4.6.4 β^--radiation

The negatively charged radiation discovered before 1900 was soon identified to be very energetic electrons. The nuclei which decay by this mode are nuclei which are above the stability line in figure 4.1 (blue grid points); these have too many neutrons (or two few protons, depending on your point of view) to be stable. The decay has the nucleus increasing Z by 1 and decreasing N by 1 and an electron emerging. Of course, when β-decay was first observed, it would be more than 30 years before the neutron was discovered and nearly 15 years before the existence of the nucleus was inferred from Rutherford scattering. So there was, for many years, no understanding of the nature of this radiation. When the neutron was discovered, it was initially assumed that a neutron could spontaneously decay into a proton and an electron. However, it was soon apparent that it was not so simple as the following questions demonstrate.

> **Question:** Can you please explain neutrinos in basic terms?
> **Answer:** Many radioactive nuclei undergo a decay called β-decay. One kind of β-decay happens if a nucleus has too many neutrons (which have no electric charge). Somehow nature knows this and takes one neutron in the nucleus and turns it into a proton; but electric charge must be conserved, so the appearance of a positively charged proton necessitates the creation of a negatively charged particle, so an electron is ejected from the nucleus. It is this electron which is the 'radioactivity'. Back in the 1930s when β-decay was first discovered a most disturbing thing was observed: the electrons which were ejected came out with a broad range of energies. This was disturbing because each electron left behind the same thing (that is a nucleus with a particular amount of energy), so the range of electron energies implied that the energy was not conserved by β-decay. This principle, energy conservation, is so dearly held by us physicists that we invented a third particle, the neutrino, which had no mass and no electric charge and carried off the right amount of energy to insure that energy was conserved. A remarkable fact is that, because it interacts so incredibly weakly with matter, the neutrino was not experimentally observed until the late 1950s. So nearly 30 years elapsed before this hypothesized particle was observed, yet no self-respecting physicist doubted its existence!

Neutrinos have been in the news lately because of the so-called 'solar neutrino problem'. We believe that we understand very well what goes on inside the Sun and yet the number of neutrinos which we observe on Earth is considerably fewer than the number predicted by our models of stars. This problem has recently been solved by measurements which find that the neutrino is not really massless but has an extremely small mass (much smaller than the electron, the lightest of ‘everyday’ particles). I will not attempt to explain how this solves the puzzle, but it does!

And the following question also asks about the effect neutrinos have on β-decay.

> **Question:** In a β-particle emission, do the β-particle and the recoiling nucleus not move along the same straight line? My friend tells me that. How can that be possible in view of momentum conservation?
>
> **Answer:** The reason your friend is right is that there is a third particle emitted in β-decay, the elusive neutrino. Therefore the residual nucleus and β-particle normally do not move along the same line. Historically, since the neutrino is so hard to detect, it was not known that there was a third particle and some people believed that, since there is a spectrum of energies for the observed β-particle, this was a violation of energy conservation. We now understand that the neutrino carries both momentum and energy.

As noted in both these answers, the spectrum of electrons observed in β-decay is not a discrete line spectrum like α-decay or atomic spectra. Rather, a continuum is observed, as shown in figure 4.4 for the β-decay of ^{210}Bi (bismuth) to ^{210}Po (polonium). Knowing masses, it can be shown that the energy released in the decay should be about 1.16 MeV, right where the spectrum of electron energy cuts off. But, the nucleus was left always in the same state (the ground state), so there is energy missing and the amount missing varies from decay to decay, depending on the electron energy of any particular decay. Where did the missing energy go? It appeared that energy conservation was violated. In 1930, even before the neutron was discovered, Wolfgang Pauli suggested that a third particle was involved in the decay; this particle he called a neutron and postulated that it had no mass but carried energy and momentum, just like a photon. Since no experiment had ever detected this radiation, it was assumed that this ‘neutron’ interacted very weakly with matter. When the ‘true’ neutron was discovered in 1932, Pauli’s neutron was renamed, by Enrico Fermi, as the neutrino, Italian for little neutron. Neutrinos turned out to be

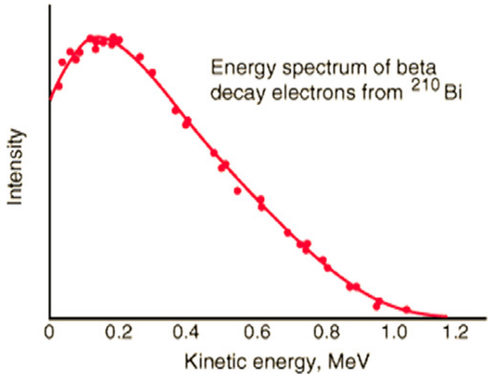

Figure 4.4. β-decay spectrum.

so weakly interacting that it was not until 1956 that they were experimentally observed by Clyde Cowan and Frederick Reines.

So the picture we take away is that, if a nucleus can become ‘more stable’ by reducing N by 1 and increasing Z by 1 (‘neutron rich’), a neutron will β-decay transforming into a proton, electron and neutrino. In fact, the neutron itself, outside the nucleus, is unstable to β-decay and has a half-life of about half an hour.

The force which mediates β-decay is called the weak interaction, the fourth fundamental force we have encountered (along with gravity, electromagnetism and the strong interaction). Neutrinos interact only via the weak interaction which is why it is so hard to detect them. Neutrino detectors work using inverse β-decay in which a neutrino is absorbed by a proton and changed to a neutron plus a positron (a β^+-particle, a positively charged electron, see section 4.6.4); the positron is then detected. Because this is such a rare event, the detectors must be huge. They are often built deep underground to shield them from background radiation like cosmic rays.

We are immersed in a huge flux of neutrinos from the Sun and the vast majority of them pass all the way through the Earth with no interactions at all. Neutrinos have recently been shown to not be massless as had been assumed for decades; their mass has not been accurately determined yet; their rest-mass energy is on the order of $mc^2 \sim 0.1$ eV, about 5 million times smaller than for an electron.

> **Question:** If neutrinos are traveling through the Earth, what distance from each other would be typical? Can they be ‘seen’ say at a light spectrum not typically seen by the human eye?
> **Answer:** The solar flux at the Earth is about 10^{11} neutrinos cm^{-2} s^{-1}, 100 billion pass through a postage stamp in one second! If you really care, you can compute the average distance between them but it is not that interesting a number, flux is what physicists care about. They are not part of the electromagnetic spectrum (like x-rays, γ-rays, radio waves, etc, are). That is, they are not photons, they are neutrinos; a neutrino interacts extremely weakly with matter whereas photons react very strongly. As a result, neutrinos are notoriously hard to detect. It was on the order of 50 years between the time we understood that neutrinos exist until the time when we actually directly observed them.

I did not answer the question about the typical distance between neutrinos, but I could have. If 10^{11} neutrinos pass through 1 cm^2 in a second and have a speed of 3×10^{10} cm s^{-1} (approximately the speed of light), then 10^{11} are contained in a volume of $(1 \text{ cm}^2) \times (3 \times 10^{10} \text{ cm s}^{-1}) \times (1 \text{ s}) = 3 \times 10^{10} \text{ cm}^3$. So, the average volume occupied by one neutrino is $(3 \times 10^{10})/10^{11} = 0.3$ cm^3/neutrino and the average distance between neutrinos is approximately $\sqrt[3]{0.3} = 0.67$ cm.

4.6.5 β^+-radiation

What about nuclei which are proton rich, those below the stability line (orange grid points) in figure 4.1? Again, nature finds a way to utilize the weak interaction but, in this instance, by turning a proton into a neutron—β^+-decay. In this decay, the nucleus emits a neutrino and a positron (β^+-particle). The positron has exactly the same mass as an electron but positive instead of negative electric charge. In fact, it is what is referred to as the antiparticle of the electron. When the two meet they annihilate, their mass completely disappears, and the energy emerges as two photons. Positrons were first observed by Carl Anderson in 1932 but not in β-decay; he observed them in cosmic rays. β^+-decay was discovered by Frédéric and Irène Joliot-Curie in 1934. Anderson won the Nobel prize in 1936, the Joliot-Curies in 1935.

Interestingly, there is a competing mechanism to β^+-decay—electron capture.

> **Question:** My question is in the nuclear equation (proton + electron → neutron). When this nuclear reaction occurs, an x-ray is emitted. Why is an x-ray emitted and not a photon of a lower energy? What would the reason be?
> **Answer:** You must be thinking about electron capture, a β-decay process which competes with β^+-decay. Here the nucleus 'grabs' one of the atomic electrons and combines it with a proton to make a neutron and a neutrino (you didn't include the neutrino). Now, think about it: there is a hole where an electron used to be in the atom, usually the K shell. An outer electron falls into that hole and makes an x-ray which follows the electron capture.

Like β^+-decay, electron capture is mediated by the weak interaction. The wave functions of the electron orbitals in the atom all penetrate into the nucleus, even though they are very small; i.e., there is a really tiny probability of finding an atomic electron inside the nucleus, but nevertheless there is a nonzero chance of one of the excess protons finding an electron inside the nucleus. Again, nature is very clever and can take the opportunity of using that negative charge to create a more stable nucleus by reducing Z by 1 and increasing N by 1, just like β^+-decay does. Electron capture was first observed by Luis Alverez in 1937.

4.6.6 γ-radiation

Recall that atoms in excited states emit electromagnetic energy (photons) when they decay to lower states. The nucleus, like the atom, has quantized energy levels and, when an excited state decays to a lower level, the result is also a photon. These photons are called γ-radiation (γ-rays). As we have seen in this chapter, nuclear energies are on the order of MeV, so γ-rays have energies much higher than atomic photons which are typically a few eV to a few tens of keV (x-rays). Energies of nuclear γ-rays have a range of approximately 0.1 MeV to a few tens of MeV. Cosmic ray γ-rays have been observed with energies around 10^{20} eV!

There is a competing mechanism which can also result in the changing of the nuclear energy from one level to a lower level. Recall that, when discussing electron capture in section 4.6.5, it was noted that atomic electrons have nonzero wave functions inside the nucleus—there is a small probability of finding an electron inside the nucleus.

> **Question:** Have you ever heard of internal conversion in radioactive decay (if not I'm fine with that)? Because a guy on the internet told me that internal

conversion happens when atomic nuclei GAIN energy yet sources I read say that it's another way that excited nuclei LOSE energy without emitting γ-rays, is he correct?
Answer: I would be a pretty poor nuclear physicist if I hadn't heard of internal conversion! It is an alternative to γ-decay of an excited nuclear state. The nucleus certainly does not gain energy since the net result (for the nucleus) is a loss of energy as it makes a transition to a lower-energy state. The energy is carried off by the kinetic energy of an atomic electron which is ejected in the process.

The mechanism competing with γ-decay is called internal conversion. Essentially, the nucleus grabs an atomic electron, usually in the K shell, and gives all the energy which would have gone to the photon to the electron. Of course, this leaves behind a hole in the K shell so the ejected electron is followed by an x-ray from an electron in a higher orbital dropping into the hole.

4.6.7 Spontaneous fission

There is one additional kind of radioactivity we should mention here because it plays an important role in history. Occasionally a very heavy nucleus will spontaneously split into two lighter nuclei. This is called fission and, in this case, spontaneous fission (SF). Such a process is very rare, but observable. For example, ^{235}U (uranium) has a half-life for spontaneous fission of 3.5×10^{17} years. (This is about 20 million times the age of the Universe!) From section 4.6.2 we can calculate the decay rate $\mathrm{d}N/\mathrm{d}t$ knowing $T_{1/2}$; doing this, I find that $|\mathrm{d}N/\mathrm{d}t| \approx 14\,\mathrm{SF\ g^{-1}\ d^{-1}}$ for ^{235}U. (Details of my calculation can be seen in appendix I.4.) So, even such an extraordinarily long half-life leads to an observable rate for a macroscopic sample. This, again, is indicative of how very small an atom is, how astonishingly many atoms there are in a mere thimbleful of matter.

Since this can happen, it must mean that energy is released. In fact, as we shall see in section 4.7, a lot of energy is released and if we could find a way to cause fission to occur at a higher rate, we could use this process to generate energy. (And, of course, we have!)

4.6.8 Effects of radiation

Question: What's the worst kind of radiation?
Answer: This is an impossible question to answer. It depends on the intensity of the energy and the damage that it will do to a biological object (like you). Let me give you a few examples:

- Neutrinos are particles which are released in certain radioactive reactions. There are a huge number which come from the Sun; about a million billion per second strike you! But they do not interact strongly with matter and just zip right through you.
- Electromagnetic radiation can interact strongly with your cells. For example, if I were to put you at the doorway of a blast furnace, the radiation would

literally cook you within seconds. This is just plain old fashioned infrared radiation which we use to broil foods and it can be very bad for you.

- The various types of nuclear radiation, alpha, beta and gamma, can do damage if intense enough. These are called ionizing radiation and do damage by knocking electrons from biological molecules (e.g. DNA).
  - α-radiation (helium nuclei) interacts very strongly but that is good in one sense since they are easily shielded against: most α-radiation can be stopped by a sheet of paper. On the other hand, alpha emitters can be dangerous if in the environment because, if ingested or inhaled, the source is right inside you so you can't shield against it.
  - β-radiation is high-energy electrons and can be dangerous if intense enough.
  - γ-radiation is very high-energy electromagnetic energy (like light, radio, infrared, x-rays, etc) and it can do serious damage if intense enough. Sometimes that is good because this is the kind of radiation usually used to irradiate cancer tumors.

- Fast neutrons are not charged, but because their mass is close to that of a proton and there is a lot of hydrogen in our bodies, they are fairly efficient at knocking protons out of the molecules they might be residing in.
- Slow neutrons might be absorbed by a nucleus and result in radioactivity inside the body.
- Another kind of electromagnetic radiation which can be very bad is ultraviolet radiation. This is the radiation responsible for sunburn and skin cancer.

4.7 Nuclear fission

4.7.1 Introduction

There is clearly energy to be had if we can take advantage of the large binding energies of nuclei. As hinted at in section 4.6.7, spontaneous fission releases a large amount of energy. The rate of spontaneous fission, though, is too low to be a useful source of energy. However, if we could induce nuclei to fission, it might be possible to obtain a much higher rate. Before we discuss the way we do this, though, it is important to understand why energy is released. Recall from section 4.4. that the binding energy B of a nucleus is the energy required to completely disassemble all the nucleons; the binding energy per nucleon is simply B/A. Figure 4.5 shows the trend of B/A for nuclei over the range $A = 2$–250. Now, suppose we choose a nucleus with $A \approx 240$ and its fission is symmetric, two nuclei with $A \approx 120$. As you can clearly see, B/A increases by about 1 MeV. Now, there are 240 nucleons participating, so there has been a release of approximately 240 MeV. Taking the mass energy of a nucleon to be about 1 GeV, that means that the fraction of mass converted to energy is about $1/1000 = 0.001 = 0.1\%$; So, if you 'burn' a kilogram of uranium this way, you will release $Mc^2 = (1.0\text{ kg})(3 \times 10^8\text{ m s}^{-1})^2/1000 \approx 10^{14}\text{ J}$. To get a feeling for how much energy this is, one kilogram of gasoline will yield about 4×10^7 J of energy—fission is more than a million times more efficient than chemistry as an energy source. But, this should come

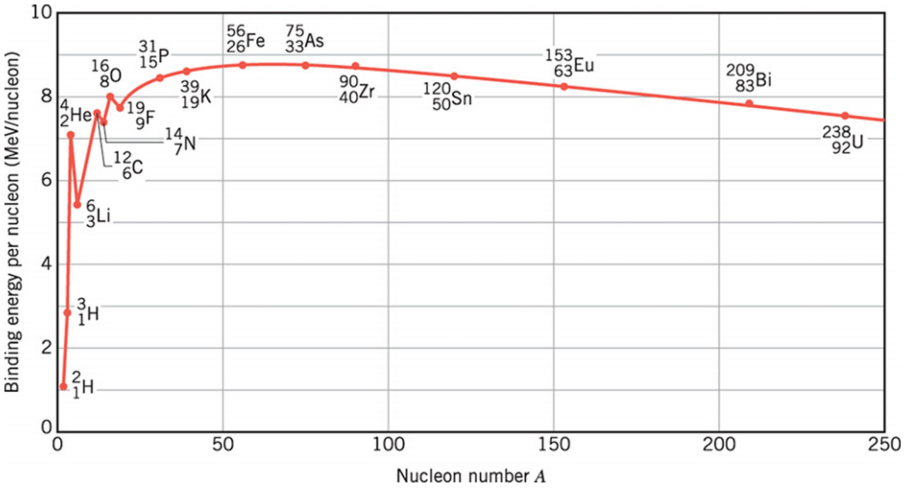

Figure 4.5. Binding energy per nucleon.

as no surprise since we have learned that atomic energies are on the order of eV whereas nuclear energies are on the order of MeV, a factor of a million larger.

The task then is to find a way to make fission happen and to control it. It would also be nice if we could make it self-sustaining, the way a fire is, so that we did not have to be continually causing it to continue.

Question: In the nuclear fission reaction ${}^{235}_{95}\mathrm{U} + n \rightarrow {}^{90}_{36}\mathrm{Kr} + {}^{143}_{56}\mathrm{Ba} + 3n$ there is no change in the mass of reactant and product so from where does the 200 MeV energy come from?

Answer: The total charge (92) and *mass number* (236) before and after this fission reaction are the same. However, this does not mean that the total mass is unchanged. If you look up all the atomic masses involved in this fission, you will find that there is less after the fission.

This question illustrates much, although it is rather brief. Let us first just look at the energetics. The mass energies are

$$\begin{aligned} m_{\mathrm{U}}c^2 &= 218938.7\ \mathrm{MeV} \\ m_{\mathrm{Kr}}c^2 &= 83758.2\ \mathrm{MeV} \\ m_{\mathrm{Ba}}c^2 &= 133127.7\ \mathrm{MeV} \\ m_{\mathrm{n}}c^2 &= 939.6\ \mathrm{MeV}. \end{aligned}$$

And so the Q-value would be $Q = m_{\mathrm{U}}c^2 + m_{\mathrm{n}}c^2 - m_{\mathrm{Kr}}c^2 - 3m_{\mathrm{n}}c^2 = 173.6\ \mathrm{MeV}$. Apparently this is the 200 MeV referred to by the questioner—173.6 MeV of energy is released which will show up as the kinetic energy of the products.

But, there are other interesting things to notice in the question. Note that a neutron is added to the uranium; this initiates the fission. The other thing to note is

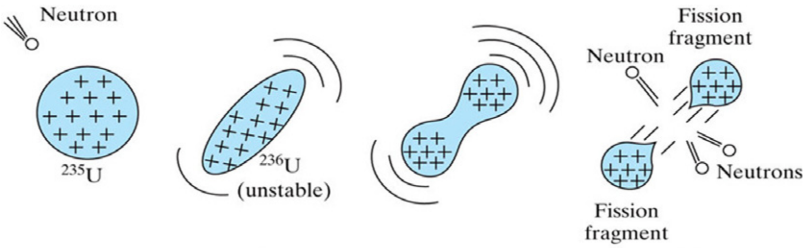

Figure 4.6. A cartoon of the fission of ^{235}U.

that there are three extra neutrons after the fission. When fission occurs the products are extremely neutron rich, so much so that they can actually emit neutrons rather than undergo β-decay. In the next two sections we will discuss how using neutrons can cause fission to occur and how the extra neutrons after the fission can be used to cause subsequent fissions.

4.7.2 How to cause fission

Nuclei are small quantum systems like the atom, lots of particles moving around under the influence of the forces they experience. But, what is the best way to envision them classically? Any classical model drawn from our everyday experience is bound to be wrong in the details, but might be useful as a guide to understanding. You might think, since the nucleons are so tightly packed, that a solid ball would be a pretty good model. In fact, that is not a useful model. Instead it is quite a good approximation to think of the nucleus as a liquid drop with a surface tension causing it to hold together. Think about such a drop floating in a gravity-free space. If you gently poke it, it will start vibrating; or if you poke it off on the side it will rotate and vibrate. So, imagine having an American football shape (mathematically called a *prolate*) and rotating about an axis perpendicular to its long axis, like the football on its side on the ground spinning. But it is a liquid and if it spins rapidly, centrifugal forces will tend to elongate it. If it then becomes too stretched out, it will form a ‘neck’ between one end and the other as shown in figure 4.6 and eventually split into two.

A convenient way to ‘poke’ the nucleus, originally done by Enrico Fermi in 1934, is to shoot neutrons at it. If the neutron adds itself to the ^{235}U nucleus it becomes a ^{236}U nucleus but in a highly excited state, a state which can be thought of as violently vibrating and/or rotating, a perfect candidate for fission. Fermi obtained his neutrons from the same kind of reaction which Chadwick and the Joliot-Curies did—by bombarding light nuclei with α-particles.

There are lots of other ways to cause a fission, you can ‘poke’ the nucleus with just about anything. But, as explained in the following sections, neutrons are the best if you want to make a reactor or a bomb.

4.7.3 How to keep fission going

In the preceding sections you should have noted that, when a fission occurs, there are several neutrons after the fission, three in the example in section 4.7.1. Imagine that these three would each cause another fission to occur. Then there would be nine new neutrons causing nine new fissions. Then there would be 27 new neutrons causing 27 new fissions. Each N such steps would result in 3^N fissions at the end and a total of $1 + 3 + 3^2 + 3^3 + \cdots + 3^N$ total fissions in the time it took to reach the Nth step. If it took $1\ \mu\text{s} = 10^{-6}$ s, a millionth of a second, for one step to happen, then a quick calculation shows that after 30 steps (which would take only 3×10^{-5} s) the total number of fissions is about 3×10^{24}. So, recalling that the typical energy per fission is about 200 MeV, the total energy released would be about $(200\ \text{MeV}) \times (1.6 \times 10^{-13}\ \text{J MeV}^{-1}) \times (3 \times 10^{24}) \approx 10^{14}$ J. The average power over that 30 μs would be about $3 \times 10^{18}\ \text{W} = 3000\,000$ TW. Compare this with the biggest power station in the world, the Three Gorges Dam in China which has a power output of about 22.5 TW. This fission is not just 'going', it is 'going wild'. This is referred to as a supercritical assembly, the power is increasing exponentially. This is the objective of a nuclear bomb. Of course, you will not get every neutron to cause another fission, but if more than one per fission causes another fission, you have a runaway situation. One would like to control the neutrons.

> **Question:** Why do we always hear about critical mass in nuclear explosions? Shouldn't it really be critical density? It seems to me that the mass of fissionable material in a bomb is always there; it's when it gets compressed to some high density that the chain reaction occurs.
> **Answer:** *Critical mass* is one of those unfortunate misnomers (because it is not a mass) which has been passed down by common usage. (Another example would be *electromotive force* which is not a force.) Your suggestion that we call it critical density is not really very good either. I would call it critical geometry. For example, a hemisphere of uranium might not be a critical mass (that is, less than one neutron from a fission causes another nucleus to fission); but, if you put two such hemispheres together, they might be critical. When the material in a bomb is 'compressed', it is brought into proximity with more material, it is not physically compressed to higher density.

As this answer shows, one way to control neutrons is to control the geometry of the material to undergo fission. If you had a thin sheet of uranium, for example, nearly all the neutrons would escape without causing another fission. You might have a sphere which was large enough that barely more than one neutron per fission resulted in a new fission; but, we could cut it in half and then the fuel would not be critical. But, putting the halves together, you would have a critical mass.

4.7.4 How to control fission

The neutrons which are released in fission are 'fast' neutrons, having kinetic energies of thousands or millions of eV. It was discovered experimentally that if they could be slowed down to eV energies (thermal neutrons), there would be a much greater chance of their being captured by the uranium nucleus.

> **Question:** Why is the probability of splitting an atom greater when sending slow neutrons rather than fast through the nucleus in fission reactors?
> **Answer:** If a neutron and a fissile nucleus interact, there is a probability, not a certainty, that a fission will occur. A slow neutron will spend much more time interacting with the nucleus and therefore the likelihood of a fission happening is greater than for a fast neutron which spends less time interacting.

A fast neutron will cause a uranium nucleus to split, but it turns out that a slow neutron has a much better chance of causing a fission. For a bomb, there is no time or space to slow neutrons down, so the fissions are caused by fast neutrons. But, in a bomb you do not want to control the fissions and in a reactor you do, so 'thermalizing' neutrons is imperative for reactors. The idea is to have a material which will slow down the neutrons without removing them. Many reactors are immersed in water which is very good because there is lots of hydrogen (protons) in water and the 'billiard-ball' effect of the cue ball stopping after a head-on collision with another ball; usually heavy water (some ^{1}H are replaced by ^{2}H) is used because a lone proton can absorb a neutron and remove it from the game. The first reactor used graphite because carbon is relatively light and easier to manage. Whatever is used is referred to as the *moderator*. More details about reactors can be found in section 4.7.5.

The moderator increases the chance of a neutron causing a fission. But, we need a way of controlling the number of neutrons. The way this is done is to have a material which can be moved in or out of the reactor and which has the property of easily absorbing neutrons. They are referred to as control rods and if inserted into the reactor, can quickly halt a reactor which has gone super critical.

4.7.5 Nuclear reactors

The first nuclear reactor was assembled under Stagg Field football stadium at the University of Chicago in 1942.

> **Question:** How did Chicago Pile 1 achieved a chain reaction? I know that they used purified graphite as a moderator and VERY pure uranium in the pile (reactor). The yellow cake was obtained from the Eldorado plant in Port Hope Ontario, which went through an ether process at the Mallinckrodt Chemical Works in St Louis. Later some of the Mallinckrodt uranium was sent to the University of Iowa at Ames to be cast. Both the cast product and the Mallinckrodt product were used in the CP-1 matrix; the cast product, being purer, being placed closest to the center. During the testing, building up to the pile, they used a beryllium/radium neutron source, both in New York City and, later, in Chicago, to test the graphite as a moderator (as well as initiators for the atomic bombs). That I understand. However, when it came to the actual pile there is no mention of a beryllium/radium neutron source. It certainly appears that they relied on the uranium itself to initiate fission. But, how did they get the first neutron(s) to begin the chain reaction? Does U-235 undergo spontaneous fission?

If so, it must be at a VERY slow rate and with a good moderator (graphite or heavy water). I've heard about spontaneous fission and the Flerov–Petrzhak discovery of spontaneous fission in 1940. Fermi must have known about this. So, did they use a radium/beryllium source or rely on spontaneous fission to start CP-1?
Answer: It is indeed true that spontaneous fission is a rare event. On the other hand, there are one heck of a lot of atoms there and even very improbable events are quite possible at reasonable rates. Indeed, the first reactor, in Chicago, had no external neutron source but relied on spontaneous fission. Spontaneous fission can also be triggered by external radiation like cosmic rays. It took me a while to find a source which explicitly said this.

The source referred to in the preceding answer may be found at www.osti.gov/accomplishments/documents/fullText/ACC0044.pdf. If you are interested in the history of nuclear energy, this is a very good first-hand account. See page 23 of the document for the statement 'With sufficient uranium in the pile, the few neutrons emitted in *a single fission that may accidentally occur* [my emphasis] strike neighboring atoms…' Recall that in appendix I it was estimated that the rate of spontaneous fission of ^{235}U is about one fission per gram every two hours; there was certainly much more than one gram in the reactor. Additional fissions are initiated by cosmic rays.

For the following question and answer, refer to figure 4.7 which shows the various parts of a reactor.

Question: One of my friends (an oncologist, so no dummy) tells me that modern nuclear power plants have a safety switch that will automatically send all the rods into a graphite core in case of emergency which will then prevent a nuclear accident, such as Chernobyl or Fukushima, and will safely contain the fissionable material essentially forever. I can't find a reference to such a safety device on the internet, although I may be just looking in the wrong place. My understanding is

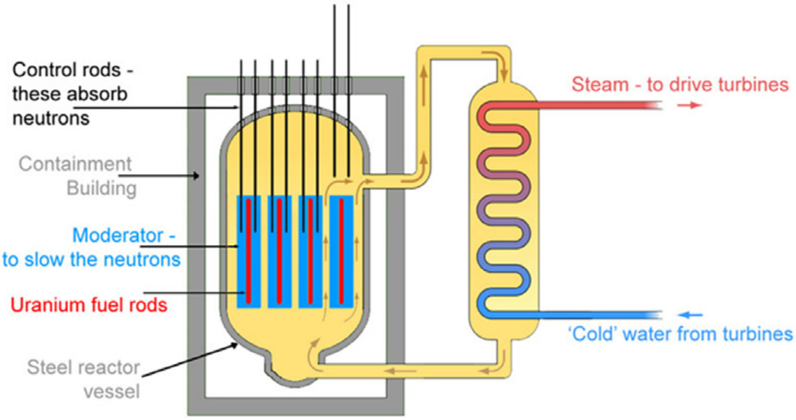

Figure 4.7. A simple nuclear reactor.

that all nuclear power plants will fail at some point and breach the containment container if water is not constantly supplied. Which is correct?
Answer: There are many kinds of reactors with many kinds of safety systems. What we can learn from earlier accidents is that we should never get cocky about their safety. That doesn't mean that they are not incredibly safe, but surely never perfect. Your friend, though, seems to have something confused. For a nuclear reactor to work, there must be an abundance of thermal (slow) neutrons to initiate the fission. However, each fission releases fast neutrons and, for the geometry of the reactor core, you could not have a self-sustained reaction without slowing them down. The purpose of graphite in a reactor is to slow down the fast neutrons (not all reactors use graphite as a moderator). So to send the fuel rods into graphite would make things worse, not better. The very first reactor, at the University of Chicago during WWII, had bars of graphite gradually added until the reactor went critical (critical means there is one new fission for each fission). The simplest reactor has a bunch of rods called control rods, often made of cadmium, which absorb neutrons efficiently; these are dropped into the reactor core in the event of an emergency to cause the reactor to go subcritical. But accidents can cause damage which cause such safety systems to be inoperative.

Follow-up question: Do nuclear power plants require a constant supply of water to remain safe? I'm not talking about the expended fuel rods, but about the core itself. Without water, will a power plant go critical at some point?
Answer: First, the terminology. 'Going critical' is not bad, it is necessary for energy to be produced. Here is how it goes:

- First with the control rods all the way in, the reactor is subcritical which means that any time a fission happens, fewer than one more occur. Each fission is like a lighted match which goes out without starting a fire.
- Now, start pulling the control rods out. Eventually you reach the point where more than one fission happens after each fission; this is called supercritical. So the rate of fission increases and you have to be careful to keep it from 'running away'.
- When it reaches the rate you want, maybe 10^{22} fissions per second, you have to stop the increasing rate by putting the control rods back just far enough so that the reactor is critical—each fission causes another so now energy is produced at a constant power of $P = E \times 10^{22}$ per second where E is the energy of a single fission. Taking $E \approx 200\,\text{MeV}$, $P \approx 3.2\,\text{GW}$.

  Now, to the crux of your question. You are producing energy, in the form of heat, at the rate of 3.2 GW which is one billion joules of energy per second. That energy has to be carried away somehow or else the whole reactor will heat up and melt (the dreaded 'meltdown'). The most common way to carry that energy away is with water and the heated water is used to drive turbines to generate electricity. If you do not do something to take the heat away, you have a disaster.

Because of the superiority of nuclear fission over chemistry in both efficiency (energy out per kilogram of fuel in) and the fact that fission does not put CO_2 into the atmosphere like other fuels do, it would seem to be the ideal source of energy for our planet. It does, though, have some serious drawbacks. The most important is that the fission products, even after emitting some neutrons, are still very neutron rich and the half-lives of these β^- and γ emitters are often thousands or millions of years. How to safely store the radioactive waste is a huge problem, one not yet satisfactorily solved. Another problem, long term, is that fissile elements are rare.

4.8 Nuclear fusion

Nuclear fission yields energy because there is an upward slope in the binding energy per nucleon curve as we go from very heavy to lighter nuclei. You can see this in figure 4.5, but also note that this upward slope ends when you reach nuclei near iron (Fe) and starts to descend. This means that for nuclei lighter than iron, energy is gained by fusing two nuclei together, not splitting one into two. And, because the slope is much steeper for light nuclei, it should be evident that fusion will be much more efficient than fission. We have noted earlier that the α-particle ($^4_2$He) is very tightly bound. Referring to figure 4.5, you can see that the α-particle has a binding energy of about 7 MeV per nucleon; a deuteron ($^2_1$He) has a binding energy of about 1 MeV per nucleon. The α-particle has two neutrons and two protons and the deuteron has one neutron and one proton, so fusing two deuterons should result in one α-particle. The energy released in such a fusion would be (6 MeV/nucleon) × (4 nucleons) = 24 MeV. The rest-mass energy of an α-particle is about 3700 MeV, so the fraction of mass turned to energy is about $24/3700 = 0.0065 = 0.65\%$. Recalling from section 4.7.1 that fission yields an energy/fuel ratio of about 0.1%, fusion is more than six times more efficient than fission.

Fusion is the process which is used in a hydrogen bomb. Much more important, though, is that nature 'knows' that fusion is the most efficient means of producing energy.

Question: At the most fundamental level, exactly where does the energy from fusion come from? I know $E = mc^2$ describes how much energy, but not the process itself. I know about the curve of nuclear binding energy. E.g., when four hydrogen nuclei fuse, the resultant helium atom has less mass and the excess is released as energy. But where exactly does the energy come from? Is it correct to say the strong nuclear force ultimately provides this? Or is simply an intrinsic process we accept ('it just happens')? At the lowest level, is there a describable mechanism by which matter stores energy, or by which the mass to energy conversion releases energy?

Answer: It does indeed come from $E = mc^2$. And, yes, it comes from the strong interaction. The example you state (4 H going to 1 He) is not a good one because it is incorrect, or at least incomplete; two of your protons must turn into neutrons + electrons which complicates things (but happens ultimately). Better to fuse two deuterons (nuclei of 'heavy hydrogen' which consist of a bound neutron and proton) into an α-particle (a He nucleus). As you correctly state, energy is released because the mass of an α-particle is smaller than the mass of two deuterons.

It comes from the process of their becoming bound together so, as you suggest, the strong force is responsible. It is perhaps easier to understand if you think of the reverse process: in order to pull apart an α-particle into two deuterons, you must supply work, right? Where does the energy that you put in go? It goes into mass.

The Sun, and all stars, convert mass into energy by fusing light elements into heavier elements. In the early Universe hydrogen was almost all there was with just a smidgen of helium and lithium. Now, at the age of about 14 billion years, many stars have converted a good deal of this hydrogen into heavier elements, but hydrogen remains the most abundant fuel for most stars in the prime of their lives. Nature is extraordinarily clever and has picked the most efficient method of energy production to power stars.

Question: Our Sun is the source of all energy in our solar system. Therefore it has great stored quantities of potential energy in the form of unfused hydrogen. I see the Sun having all the properties of a bomb. It's ignited and it has lots of unexploded fuel. What prevents it from going off all at once? Or, is it more plausible to think of the Sun, much like a smoldering log, burning slowly. The log's rate of conflagration is governed by its access to oxygen.
Answer: The Sun is, as you say, a huge ball of hydrogen. It is very hot, so the Sun is a plasma with many electrons and protons running around very rapidly. However, the process is not just a simple one-step process as in your burning log where we essentially have carbon combining with oxygen to produce CO_2. What happens is first two protons (^1H) fuse and β-decay to heavy hydrogen ^2H; then the ^2H fuses with a ^1H to make ^3H; then the ^3H β-decays to ^3He; then ^3He either fuses with a ^1H and β-decays to make ^4He or else two ^3He fuse to make one ^4He and two ^1H. But for each step to happen you have to have the right parts come together which happens with a probability determined by many things; and even if the parts come together, there is only some probability that the desired reaction will happen. So it doesn't all happen at once because of the limiting probabilities governing things. A nice place to read about this so-called proton–proton cycle is here. Incidentally, the H-bomb does not work like the Sun (but, of course, does use fusion) and you want it to go off all at once; therefore it is designed as a single-step process of fusing two ^2H to make ^4He.

So, the natural question to ask is 'why are we not using this wonderfully efficient source of energy on Earth to power our civilization?'

Question: Why is fusion so hard to crack and when can we expect fusion to replace coal and oil?
Answer: Fission is fairly easy because all you have to do is 'tickle' a heavy nucleus and it will split in two; this is usually achieved by adding a slow neutron to the nucleus which is easy to do because it has no electrical charge and therefore does not feel any repulsive force from the nucleus. In addition, the Coulomb force

> worked to our advantage because once the fission has 'necked', the electrical repulsion between the two halves helped push the fission to conclusion. Fusion, however, involves bringing two positively charged light nuclei together. Since they repel each other, they can only come close together if they are going very fast. Another way of saying the same thing is that the fusing target material (generally isotopes of hydrogen) must be very hot. Containing a hot enough gas (plasma, actually, since the high temperature will cause the atoms to be ionized) has turned out to be an extraordinarily difficult engineering problem. There is an old saying among physicists, tongue in cheek, that 'fusion is the energy of tomorrow and it always will be!' Scientists and engineers have been working for more than 50 years to try to build a reactor, with considerable progress but no ultimate success.

A star continues, during its lifetime, to fuse its contents into heavier and heavier products, but stops at iron (Fe) because beyond iron it costs energy to go further; again, refer to figure 4.5 to see that iron has the highest binding energy per nucleon.

> **Question:** I am a university senior student. My department is elementary science education. I know that every atom that is in our planet was once created in stars. I know that first H is used to create He and all the way to iron. However, iron is not the last element in the periodic table, how were the other elements formed?
> **Answer:** As you note, iron is the heaviest element created in stars; the reason is that iron is the most tightly bound nucleus and therefore any heavier elements will require that energy be added rather than energy being released by fusion (which is what stars do). Heavier elements are created in very energetic events like novae and supernovae. Recently it has been found that collisions between neutron stars also play an important role, particularly for the element gold.

Since the early Universe had, for all intents and purposes, only hydrogen in any abundance, there were originally no planets, no elements heavier than lithium, only stars. But, as indicated above, heavier elements were manufactured in stars in the early Universe and these stars added the richness of elements necessary for our biological lives to be possible. Carl Sagan said, in his book *The Cosmic Connection: An Extraterrestrial Perspective*, 'Our Sun is a second- or third-generation star. All of the rocky and metallic material we stand on, the iron in our blood, the calcium in our teeth, the carbon in our genes were produced billions of years ago in the interior of a red giant star. We are made of star-stuff.'

4.9 The standard model

So, we have learned that the atom is made of protons, electrons, and neutrons. Would it not seem natural to ask 'what are they made of?' This is what is addressed by the standard model of elementary particles. The picture is that all nature is composed of three kinds of elementary particles, field quanta, quarks and leptons. The whole picture is beyond what we will address in this book, but a broad overview is fairly straightforward. The standard model is only for three of the four fundamental forces—electromagnetic, strong nuclear and weak nuclear; gravity seems to not fit in and there is no successful theory of quantum gravity.

Refer to figure 4.8 for the following discussion.

There are six quarks (purple in figure 4.8) (u, d, c, s, t, b), the components of hadrons which include neutrons and protons. All quarks have electric charge ±1/3 or ±2/3 so they feel the electromagnetic force. They also feel the strong force. They also feel the weak force since we know that β-decay changes a neutron/proton to a proton/neutron via the weak interaction. All are fermions.

There are six leptons (green in figure 4.8) (electrons (e), muons (μ), and taus (τ), and three associated neutrinos). All feel the weak force. The neutrinos have no electric charge, so they do not feel the electromagnetic force; the charged leptons, of course, do. All are fermions.

For each force there is a 'field quantum', that particle which communicates the force between particles which feel that force (the pink in figure 4.8—γ, g, Z^0, $W^{\pm}$); note that these are all bosons since they have spin 1. For the electromagnetic force, it is the photon. For the strong force, it is the gluon. For the weak force, it is the Z and the W bosons. Note that only the Z has electric charge. The electromagnetic and weak forces have been unified in theory, sometimes collectively called the electroweak force.

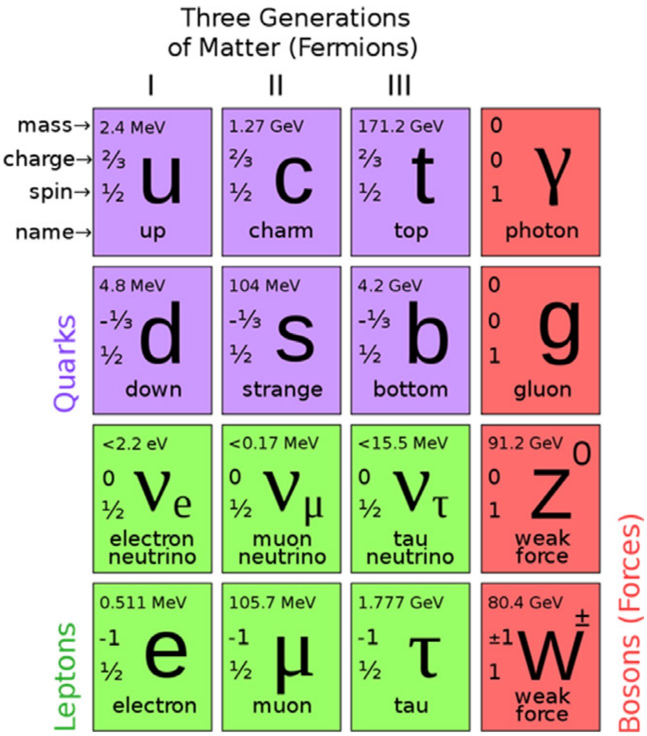

Figure 4.8. Elementary particles of the standard model.

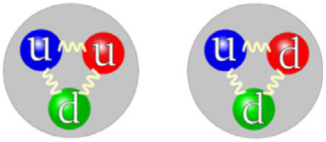

Figure 4.9. Quark structure of a proton (left) and neutron (right).

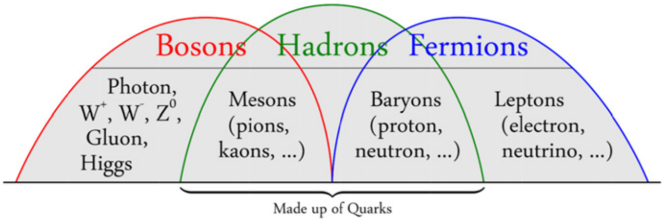

Figure 4.10. Families of particles.

Question: What is in a neutron that makes it different from a proton? Is it the stuff that makes up an electron?
Answer: It has nothing to do with electrons. Neutrons and protons, often referred to collectively as nucleons, differ in their internal structure. Shown in figure 4.9 are a proton which has two ‘up’ quarks and one ‘down’ quark (left) and a neutron which has two ‘down’ quarks and one ‘up’ quark (right). Particles composed of quarks are called hadrons and also include mesons.

The wiggly white lines in figure 4.9 represent gluons. You may conclude that what happens in β-decay is that quarks change to different quarks plus a pair of leptons.

Finally, a Q&A which gives an overview of all known particles.

Question: So I understand that protons and neutrons are made of quarks. A proton is made of two up quarks and one down quark and a neutron is made from two down quarks and one up quark. I know that much. But there are so many other particles in the standard model. Are there any other large particles we can (or cannot) observe besides protons and neutrons that are composed of quarks or bosons or whatever? I have been wondering this for a while.
Answer: There is, of course, no concise answer to your question. The standard model has numerous particles and some are made of quarks (hadrons) and some are not (leptons and field quanta). The hadrons which are bosons (integral spin) are made of two quarks, hadrons which are fermions (half odd integral spins) are made of three quarks. There are also many ‘particles’ called resonances which are excited states of the more fundamental hadrons. Figure 4.10, which I borrowed from Wikipedia, gives a pretty good overview, I think. For more details, read the Wikipedia article on the standard model.

Chapter 5

Epilogue

5.1 Ask the psychic

It is amazing how many visitors to the *Ask the Physicist* web site think they are asking the psychic. In spite of the disclaimers I have put on the home page, nearly every day I receive at least one question asking me to use my mystical powers to help guide their daily lives or explain remarkable phenomena. And, it is not just ignorant people who cannot spell—computer algorithms don't seem to get it either. A few years ago I tried to use GoogleAds on the site to generate a little money; to my horror, many ads generated by Google's software were for 'Free Psychic Readings'! And, if you have ever tried to talk to a human being at Google, forget it; I just gave up using GoogleAds when I couldn't stop the psychic ads—what better way to destroy the credibility of a serious science site than with ads for palmists and clairvoyants? Gathered here are some of my favorites from Ask the Psychic. (By the way, unlike the serious parts of this book, questions in this and the next section are not edited.)

> **Question:** Will i lose my house when i go to courtQ?
> **Question:** Is my boyfriend lozzario smith beat this charge hes facing?
> **Question:** Will Dillon Gilbert Johns like me more than a girl in my class name Charli?
> **Question:** Will Hudson Catholic win the county championship in basketball tomorrow??
> **Question:** Where are the church keys that Melodie thinks I stole, but didn't?
> **Question:** I know its just another love story but im struggling. My ex left me for his his best friend (female) tells me all these things about how she's a better person & that hurt. We dated over a year+ & i thought i was okay.but i have been dreaming of him treating her better then other dreams we are back together.i wake up & i feel like we are back together then get crushed remembering How horrible things went down. Any advice how to help my mind..help me?

doi:10.1088/978-1-6270-5939-8ch5

> **Question:** I have 2kids with my ex gf she left me 3 times for 3different guys i know its over but some happed to me and whant to be there for bad but I didn't let idk wat to do about her I keep having dreams about her having a baby by her new bf?
> **Question:** I recently had a dream that there will be a flood that cover the Earth. Two years ago I had a dream I was on what appeared to be one of nine large ships because of a flood. Is it possible to save all mankind and do we have the resource to house everyone on floating houses?
> **Question:** Is my husband going to die soon?
> **Question:** I purchased a Breast Enhancing Cream, will it work for me?
> **Question:** My name is Alanna, I am 16 years old, & I really care about this girl name Jehosheba. Me & her we're dating for a month and one day. But before we made it official we were talking for a very long time. She broke up with me not to long ago and I miss her a lot. I keep wondering will she ever hit me up ? Was it ever real ? Did she ever really like me ? I really miss her will we ever get back together ? She told me she loves me and she would NEVER leave me
> **Question:** when will my careerr land and have a good steady job when will I have a boyfriend when will I be pregnant how many children will I have and what sex what names would my children have
> **Question:** So i'm just going to say it right out! For a few years ago, i started to dream about a girl, that i do not know, and have ever seen before, and the dream is super realistic it's crazy. And i have always had these dreams, were we met and talked, went out and eat and stuff like that. but i have never really seen her face, there have always been like a cloud in front of her face, like i can't explain it. And i remember one thing that she said to me a few months ago, she said to me *i'am always here for u Thomas*. I'm I going crazy or something bc i feel like i know her, but in the same time i do not! I hope u can help me out ant explain whats going on in my head !

And, again, the perennial favorite,

> **Question:** Is I'm pregnant

5.2 Off the wall hall of fame

Although I emphasize that I require single, concise, well-focused questions, inevitably I receive lengthy questions (dissertations, really) from people wishing to get the stamp of approval of a professional physicist on their personal theory of something or other. If these submissions are questions rather than assertions, then they are often questions which start with something like 'is it possible that…' Of course a scientist is often loathe to say that something is impossible even if he believes it to be—then you are attacked for being closed-minded. Best to just file these away in my Hall of Fame rather than try to enter into a dialog with a crazy person! Here are a few good ones.

Question: If energy is conserved, why is there energy crisis?

Question: Hello, I have been talking to Nikola Tesla, with not too much success, although I think answers come from other sources. My question is difficult to put in a nutshell. Its about HAARP, Cell phone towers, smart meters, all the bad RF Radiation, being frequency. Is it possible to have something that would put out a certain frequency to bounce back, change/alter/make good, or STOP in its tracks, bad frequency? I am not university educated and especially not anywhere near enough aware or learned in sciences, but this is a very important question of the day. Is there anyone you know today working on this kind of thing? or do we still look to Tesla, since today i learned he communicated with Mars. Some peope are just so way ahead of their time, it makes you wonder how? There is so much more than meets the eye.

Question: Theory: E = mc*2 has a symmetrical pairing… AE = amc*2. I theorize that the antimmater annihilated in the 'big bang'/primordial universe produced a symmetrical explosion of anti-energy or dark energy. This would seem to account for the prevalance of dark energy and dark matter in the observable Universe; the missing mass holding galaxies together; and the inexplicable expansion rate of the Universe. Question: is it possible that antimatter has properties we are unable to directly observe and quantify…. but can indirectly observe the effects of? IS it possible that the dark energy and dark matter are in fact… merely the residue of the big bang's ANTI-MATTER explosion? according to established theory…. only some 3% of the mass survived; the other 97% being annihilated with its antimatter counterparts. My theory is radical and likely to be wrong. But before discarding it…. i ask you to explain to me WHY.

Question: Hi! I have this theory through which we might be able to, or at least find a more plausible answer, to the mystery of interdimensional/time travel. I think since going to another Universe or another time frame, for example 10 s in the past, would violate the rule of physics since there will be new energy introduced to the Universe at that time frame, and since our Universe is an isolated system. There will be a disparency. Two of the following outcomes might take place, either the whole Universe will collapse in matter of seconds and the whole Universe might just disappear or there will be a segregation and each and every person will be locked in a 'box-dimension' with their version from the past and the future time frames, and be trapped in the infinity paradox, not the usual ones but a paradox where you are dead, alive, old, young, sick, healthy, depressed, exicited, and anything that a human can encounter in his lifetime at the same time and you are seeing this, interacting with this, the part where infinity comes in as a verion of you dies, another is born. So what exactly happens when you introduce alien/outside source of energy in an isolated system?

Question: Im sorry for this being long, but i hope you will take the time to read it and see if you can answer my question. What if our Universe is one multiverse that lives along side so many others. And what if the energy that surrounds these multiverses was like an electron cloud and the multiverses were electrons of some type of atom. Then each electron or multiverse would be made up of even smaller energy sources or as i think of it multiple galaxies that build up to be an electron. Then those energies or galaxies would have to be made up of even smaller energy sources such as stars and planets. What i am trying to say is, what if our solar system is just another little energy source that takes part in an even bigger energy source that builds up electrons or multiverses. Each electron might look the same but would also be verry different in some sort of way. What if maybe the multiverses are the electrons that build up an atom. Knowing that you can not destroy an atom or create one, what if some type of energy like a big bang is creating the atom's electrons. Then they would be gathered up to form one atom. Or maybe what if the multiverses are the nucleus of what ever atom it is making. Knowing that the nucleus is made up of protons and neutrons and even those are made up of smaller energies that are called quarks. The galaxies could be the quarks to the multiverses being the nucleus of a much bigger energy that would be an atom. That would mean quarks would have to be made up of even smaller energies and thats where the stars of the galaxies come in. Since space is infinite, wouldnt the matter around us also be infinite? If so that would mean our atoms are even smaller than the energies that make up a quark and would keep on getting smaller and smaller as you look deeper into it. Since there is no way to create or destroy an atom, wouldnt that make it infinte? If so then that would mean there are much smaller energy sources than we know of. And we are just a source that cant be seen like an atom but we are much smaller. The cycle would keep on repeating, there would be atoms much smaller than atoms making up electrons that are needed to make an atom. And the big bang would be the source that starts the whole cycle.i am not referring to the big bang of our Universe, i am referring to the big bang of all the multiverses that are still being made even though we can not see it happen, much like an atom. Could this be a possibility?

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Appendix A

The constants of electricity and magnetism

Most textbooks introduce the unit of charge (coulomb) before the unit of current (ampere). I have done it a little differently here because the thing which is actually operationally defined is the ampere. Since this book does almost no electricity and magnetism, there is no real reason to introduce the coulomb first. In addition, most laypersons have a much better idea what an ampere is than what a coulomb is.

Two long parallel wires, each of length L and separated by a distance r carry electric currents I_1 and I_2. They are observed to exert equal and opposite forces (Newton's third law) on each other and the magnitude of this force is proportional to the each of the currents, the length of the wires and inversely proportional to the separation: $F \propto LI_1I_2/r$. Choosing a proportionality constant $\mu_0/(4\pi) = 10^{-7}$ defines what the unit of electric current is: $F/L = \mu_0 I_1 I_2/(2\pi r)$. So, if two wires are carrying equal currents, are separated by 1 m and the force per meter each wire experiences is $2 \times 10^{-7}\ \mathrm{N\,m^{-1}}$, then each wire is carrying an electric current of 1 ampere (A). Since electric current is the rate at which electric charge is flowing, knowing the ampere also lets us know the electric charge unit we will use, called the coulomb (C) because $1\ \mathrm{A} = 1\ \mathrm{C\,s^{-1}}$. To set the scale relative to everyday life, 1 A is a typical household current. The charge on an electron is -1.6×10^{-19} C, so a current of 1 A corresponds to $1/1.6 \times 10^{-19} = 6.25 \times 10^{18}$ electrons per second. So, the first constant μ_0, called the permeability of free space and which sets the scale of magnetic fields in the system of units we use, is exactly (because we defined it that way)

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N\,A^{-2}} = 4\pi \times 10^{-7}\ \mathrm{N \cdot s^2\,C^{-2}}.$$

We now know what a coulomb is. If we go to a laboratory and measure the force F between two electric charges, Q_1 and Q_2, separated by some distance r we find that $F \propto Q_1Q_2/r^2$. Now, to make this an equation we need to measure the proportionality constant because we know how charge and length are measured. Doing this, we find that $F = Q_1Q_2/(4\pi\varepsilon_0 r^2)$; this is called Coulomb's law. Note that we have chosen to write the proportionality constant (which we have measured) as $1/(4\pi\varepsilon_0)$. So, the

doi:10.1088/978-1-6270-5939-8ch6

second constant ε_0, called the permittivity of free space and which sets the scale of electric fields in the system of units we use, is exactly (because we measured it)

$$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C}^2\,(\mathrm{N}\cdot\mathrm{m}^2)^{-1}.$$

Maxwell's equations predict waves which have a velocity of $1/\sqrt{(\varepsilon_0\mu_0)} = 3\times10^8\,\mathrm{m\,s}^{-1}$. This is truly one of mankind's most remarkable intellectual achievements!

Appendix B

Energy

B.1 Work-energy theorem in Newtonian mechanics

Although I use calculus so that I have generalized to forces which might vary, I do the calculations in one dimension for clarity. First, derive the work-energy theorem in Newtonian physics:

$$\begin{aligned}F &= \mathrm{d}p/\mathrm{d}t \\ &= m(\mathrm{d}v/\mathrm{d}t) \\ &= m(\mathrm{d}v/\mathrm{d}x)(\mathrm{d}x/\mathrm{d}t) \\ &= m(\mathrm{d}v/\mathrm{d}x)v.\end{aligned}$$

Rearranging,

$$F\mathrm{d}x = mv\mathrm{d}v.$$

Integrating,

$$W = \int F\mathrm{d}x = m\int v\mathrm{d}v = \tfrac{1}{2}m{v_2}^2 - \tfrac{1}{2}m{v_1}^2 = \Delta K.$$

B.2 Potential energy

Suppose there is some force, call it the *internal* force F_{int}, which is always present for some particular problem and that all other forces doing work are represented by F. Then

$$\begin{aligned}W = \int F\mathrm{d}x + \int F_{\mathrm{int}}\mathrm{d}x = W_{\mathrm{ext}} + \int F_{\mathrm{int}}\mathrm{d}x = \Delta K \\ \Delta K - \int F_{\mathrm{int}}\mathrm{d}x = W_{\mathrm{ext}}.\end{aligned}$$

doi:10.1088/978-1-6270-5939-8ch7

Now, the indefinite integral $-\int F_1 \mathrm{d}x$ is just some function of x, call it $U(x)$, so

$$-\int F_1 \mathrm{d}x = U(x_2) - U(x_1) = \Delta U(x).$$

U is called the potential energy. So now we define the total energy as $E = K + U$ and rewrite the work-energy theorem as

$$\Delta E = W_{\mathrm{ext}}$$

where W_{ext} is the work done by external forces which are all forces for which a potential energy function has not been included in the energy. So, you see, the potential energy is really just a clever bookkeeping device to keep track of work done by a force which is always present.

In this book, you will seldom need to understand potential energy. The one simple case which you might need is the gravitational potential energy. In that case the force is $F_1 = -mg$ (the $-$ sign because it points down) and so $U_{\mathrm{grav}}(y) = mgy$. (We usually use y for the vertical direction rather than x.)

B.3 Energy in special relativity

In the theory of special relativity everything is the same except p is redefined:

$$\begin{aligned} F &= \mathrm{d}p/\mathrm{d}t \\ &= m\left(\mathrm{d}\left[v/\sqrt{\left(1 - v^2/c^2\right)}\right]/\mathrm{d}t\right) \\ &= m\left(\mathrm{d}\left[v/\sqrt{\left(1 - v^2/c^2\right)}\right]/\mathrm{d}x\right)(\mathrm{d}x/\mathrm{d}t) \\ &= mv\left(\mathrm{d}\left[v/\sqrt{\left(1 - v^2/c^2\right)}\right]/\mathrm{d}x\right) \\ &= mv\left(1 - v^2/c^2\right)^{-3/2}\mathrm{d}v/\mathrm{d}x. \end{aligned}$$

Rearranging,

$$F\mathrm{d}x = mv\left(1 - v^2/c^2\right)^{-3/2}\mathrm{d}v.$$

Integrating,

$$W = \int F\mathrm{d}x = m\int v\left(1 - v^2/c^2\right)^{-3/2}\mathrm{d}v = mc^2(\gamma_2 - \gamma_1) = \Delta K.$$

If the particle started from rest, $\gamma_1 = 1$ and ended at speed v, $\gamma_2 = \gamma = 1/\sqrt{(1 - v^2/c^2)}]$, then

$$K = mc^2(\gamma - 1).$$

Be sure to note that m is the rest mass. Now, this does not look much like $\frac{1}{2}mv^2$ for small v, so we need to look a little more closely. If $v \ll c$,

$$1/\sqrt{\left(1 - v^2/c^2\right)} \approx 1 + \tfrac{1}{2}v^2/c^2 + \cdots$$

This is just a binomial expansion, $(1+z)^n \approx 1 + nz + ½n(n-1)z^2 + \cdots$ So now we can write

$$K \approx mc^2\left(1 + ½v^2/c^2 + \cdots - 1\right) \approx ½mv^2.$$

Rearranging the equation for K above,

$$K = \gamma mc^2 - mc^2.$$

This equation says that 'the kinetic energy is something minus some constant'. We interpret this to mean that the 'something' is the total energy E and the constant mc^2 is energy something has by virtue of its mass, even if at rest. So, we can finally write the total energy as $E = \gamma mc^2$. So, if the particle is at rest, $\gamma = 1$ and $E = mc^2$. Although I will not work it out, a little algebra leads to the very useful expression for the total energy in terms of the momentum:

$$E = \gamma mc^2 = \sqrt{\left(p^2c^2 + m^2c^4\right)}.$$

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Appendix C

Elastic collisions

C.1 Classical collisions

First consider the one-dimensional (all velocities in a straight line) collision between two nonrelativistic ($v \ll c$) masses for which linear momenta may be written as mv and kinetic energies as $\frac{1}{2}mv^2$. A mass m_1 comes in with velocity v_1 and collides elastically with a mass m_2 at rest; after the collision, m_1 has a velocity u_1 and m_2 has a velocity u_2. All this is shown in figure C.1. Now, momentum conservation and energy conservation may be written as

$$m_1 v_1 = m_1 u_1 + m_2 u_2$$
$$\frac{1}{2}m_1 {v_1}^2 = \frac{1}{2}m_1 {u_1}^2 + \frac{1}{2}m_2 {u_2}^2.$$

There are two equations with two unknowns. Solving them,

$$u_1 = \left[(m_1 - m_2)/(m_1 + m_2)\right]v_1$$
$$u_2 = \left[2m_1/(m_1 - m_2)\right]v_1.$$

It is interesting that, since one of the equations is quadratic, there is a second solution to the problem, $u_1 = v_1$ and $u_2 = 0$—nothing happened! Note that if $m_1 = m_2$, the incoming ball stops dead and the at-rest ball leaves with the same speed as the

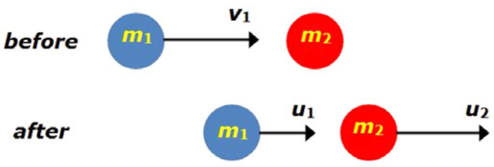

Figure C.1. A nonrelativistic, elastic, one-dimensional collision.

doi:10.1088/978-1-6270-5939-8ch8

first came in with—try it with billiard balls sometime. Also, if $m_1 < m_2$, then $u_1 < 0$ which means that the incoming ball rebounds in the opposite direction.

C.2 Relativistic collisions

Since our focus here is on Compton scattering, the two-dimensional analysis of photon scattering from electrons will be presented. Relativistically, the equations of momentum conservation and energy conservation may be written as

$$p_{\gamma 1} = p_{\gamma 2} \times \cos\theta + p_{e2} \times \cos\varphi$$

$$0 = p_{\gamma 2} \times \sin\theta - p_{e2} \times \sin\varphi$$

$$E_{\gamma 1} + m_e c^2 = E_{\gamma 2} + \sqrt{\left[\left(m_e c^2\right)^2 + {p_{e2}}^2 c^2\right]}$$

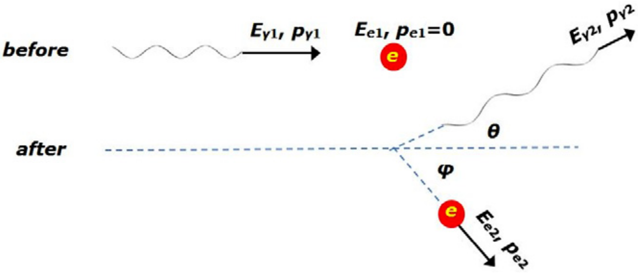

Figure C.2. Compton scattering.

where $E_{\gamma i} = hf_i = hc/\lambda_i = p_{\gamma i}c$ where f_i and λ_i are the frequencies and wavelengths of the photons. The geometry is shown in figure C.2. This is not a trivial problem, although it is just algebra. I give only the final results for the change in wavelength of the photon: $\lambda_2 - \lambda_1 = [h/(mc)](1 - \cos\theta)$. If you are interested in the details of the algebra, there are four unknowns here, $p_{\gamma 2}$, p_{e2}, θ and φ and only three equations, so the best we can do is eliminate the two variables p_{e2} and φ. (Keep in mind that λ_2 is just another way of rewriting $p_{\gamma 2}$.)

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Appendix D

Commonly used units

This appendix should be read after reading appendix A which specified how the units of electric current (the ampere, A) and electric charge (the coulomb, C) are defined.

When studying microscopic systems like atoms and nuclei, use of the usual SI units can be cumbersome because quantities are so small. The size of the atom is on the order of 10^{-10} m and the size of a nucleus is on the order of 10^{-15} m; therefore we will often use the nanometer (1 nm = 10^{-9} m) or femtometer (1 fm = 10^{-15} m) for length.

Similarly, masses are small; an electron mass is about 9.1×10^{-31} kg and a proton or neutron (nucleons) mass is about 1.67×10^{-27} kg. Usually, though, we will express the mass of microscopic particles by specifying instead their rest-mass energies (mc^2); so, $m_{\text{electron}}c^2 \approx 8.19 \times 10^{-14}$ J and $m_{\text{nucleon}}c^2 \approx 1.50 \times 10^{-10}$ J. But joules are clearly also cumbersome because the rest-mass energies of subatomic particles are so small. Rather than work with nJ or fJ, though, a new unit of energy is introduced, the *electron-volt* (eV). The eV is the kinetic energy that an electron acquires when it accelerates across the potential difference of one volt (V). You may not know what potential difference is, but the potential difference across the ends of a flashlight battery is about 1.5 V. If you accelerate a charge of 1 C across 1 V you obtain 1 J of kinetic energy; therefore if you accelerate one electron (1.6×10^{-19} C) across 1 V you obtain a kinetic energy $K = 1.6 \times 10^{-19}$ J = 1 eV. So, the rest-mass energy of an electron is

- $m_{\text{electron}}c^2 \approx 8.19 \times 10^{-14}\ \text{J}/1.6 \times 10^{-19}\ \text{J eV}^{-1} \approx 5.1 \times 10^5\ \text{eV} = 0.51$ MeV and
- $m_{\text{nucleon}}c^2 \approx 1.5 \times 10^{-10}\ \text{J}/1.6 \times 10^{-19}\ \text{J eV}^{-1} \approx 9.4 \times 10^8\ \text{eV} = 0.94$ GeV.

(The unit prefixes G (giga) and M (mega) mean 10^9 and 10^6, respectively.) So energies will usually be expressed in eV, keV, MeV or GeV. Often masses are given in units of MeV c^{-2}.

We will often encounter frequencies, as in cycles per second which have the units of s^{-1}. It is common to specify frequency in hertz (Hz) where 1 Hz = 1 s^{-1}.

doi:10.1088/978-1-6270-5939-8ch9

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Appendix E

The Schrödinger equation

This appendix is included for completeness and is in no way required to understand the text. The more advanced reader may find this material instructive.

I will do the time-independent Schrödinger equation because it is easiest to understand without much mathematics. Also, I will do the one-dimensional version; think of a bead moving on a wire or perhaps a mass attached to a spring and bouncing back and forth.

First, do classical physics: total energy E is the sum of kinetic energy T (energy by virtue of motion) plus potential energy $V(x)$ (energy by virtue of position in some force field), $T + V(x) = E$. The kinetic energy is $T = \frac{1}{2}mv^2$ where m is the mass of the object and v is its speed. But, since linear momentum p is $p = mv$, kinetic energy is often written as $T = p^2/(2m)$, so $[p^2/(2m)] + V(x) = E$. Schrödinger's equation is nothing more than the quantum mechanical equivalent of this almost intuitive (the total is the sum of its parts) energy equation.

So, what is different about quantum mechanics? It turns out that some quantities in nature must be treated as mathematical operators, not as just simple numbers like they are in classical mechanics. This is the case for linear momentum which becomes $p = -\mathrm{i}\hbar(\mathrm{d}/\mathrm{d}x)$ where $\mathrm{i} = \sqrt{-1}$, the imaginary unit, $\hbar = h/(2\pi)$ where h is a fundamental constant called Planck's constant, and $(\mathrm{d}/\mathrm{d}x)$ is the derivative with respect to x. If you have never studied calculus, a derivative is an operator which, when it operates on a mathematical function, tells you how fast that function is changing as x changes. Planck's constant is an extremely small number which is why quantum mechanics is not noticeable except for in very tiny systems (think atoms, nuclei, etc).

Now, of necessity, things become a little more mathematical; but if you get a little lost here, just carry on. We still might want to know what the linear momentum is, not just what the operator associated with it is. To do this we must introduce what is called an eigenvalue equation which is of the form $\mathbf{O}\Phi(x) = O\Phi(x)$. $\mathbf{O}$ is the operator, $\Phi(x)$ is called the eigenfunction and O is called the eigenvalue. The eigenfunction contains the information about what the system we are interested

doi:10.1088/978-1-6270-5939-8ch10

in is, and the eigenvalue is what the observable quantity associated with the operator is. For example, if it is linear momentum we are interested in, then $-i\hbar[d\Phi(x)/dx] = p\Phi(x)$ where p is the value of the momentum you would measure.

Back to Schrödinger's equation: we still use

$$\left[p^2/(2m)\right] + V(x) = E$$

but now

$$p^2 = \left[-i\hbar(d/dx)\right]^2 = -\hbar^2\left(d^2/dx^2\right).$$

(d^2/dx^2 means the derivative of the derivative, but that is not so important, just math.) So now the total energy of a system is an operator, not a number. This operator is called the Hamiltonian,

$$H = \left[-(\hbar^2/(2\mathrm{m})\right]\left(d^2/dx^2\right) + V(x).$$

To find the energy of a system, therefore, there must be an eigenvalue equation; this is Schrödinger's equation

$$\left[-\hbar^2/(2m)\right]\left(d^2\psi(x)/dx^2\right) + V(x)\psi(x) = E\psi(x).$$

The eigenfunction $\psi(x)$ is called the wavefunction and has a very special meaning. It has no particular physical meaning on its own but, when squared, it becomes the probability density function. This means that $\psi(x) \times \psi(x)dx$ is the probability of finding the particle between the positions x and $x + dx$ where dx is some very small change in x. Of course, if $\psi(x) \times \psi(x)$ is to have this meaning, the sum of all probabilities over all space must add up to 1 (the particle has to be somewhere).

From here on it is a matter of applying this equation to a particular system to find out what energies will satisfy the equation. A one-dimensional example is a mass on a spring. When you put in the appropriate potential energy function you find that the spring cannot have just any old energy, there are only certain energies which will satisfy the Schrödinger equation; these are $E_n = (n + \frac{1}{2})hf$ where n is any positive integer or zero and f is the frequency which the particular spring oscillates with. Another example is an atom. Here, of course, we have a three-dimensional problem and have to use a three-dimensional Schrödinger equation which would have the form

$$\left[-\left(\hbar^2/(2m)\right]\left[\left(d^2\psi(x, y, z)/dx^2\right) + \left(d^2\psi(x, y, z)/dy^2\right) + \left(d^2\psi(x, y, z)/dz^2\right)\right]$$
$$+ V(x, y, z)\psi(x, y, z) = E\psi(x, y, z).$$

What you find is still that an atom can only exist in certain energy states, not any old energy you might want.

Appendix F

The Bohr model of hydrogen

The force F_e between a proton and an electron separated by a distance r is

$$F_e = ke^2/r^2$$

where $k = 1/(4\pi\varepsilon_0) = 9 \times 10^9\ \mathrm{N \cdot m^2\ C^{-2}}$. Taking the proton as having infinite mass, if the electron is in a circular orbit of radius r with a speed v, then

$$F_e = ke^2/r^2 = mv^2/r$$

where $m = 9 \times 10^{-31}$ kg is the electron mass. (Note that this is Newton's second law since the centripetal acceleration is v^2/r.) So, the velocity of the electron in its orbit is

$$v = \sqrt{\left[ke^2/(mr)\right]}.$$

The potential energy of the electron a distance r from the proton is

$$V(r) = -ke^2/r$$

and the total energy is therefore

$$E = \tfrac{1}{2}mv^2 - ke^2/r = -ke^2/(2r).$$

Note that the potential energy is chosen to be zero at $r = \infty$. The fact that the energy is negative implies that this is a bound system; or, a negative energy means that if you do positive work on the system (add energy), you can move it to where $E = 0$.

This is all classical mechanics and classical electromagnetism. Bohr now introduces a quantization condition, that the angular momentum (mvr) be an integral multiple of $\hbar = h/(2\pi)$, $mvr_n = n\hbar$ where $n = 1,2,3\ldots$ The electron can only be in these allowed orbits of radii

$$r_n = n\hbar/(mv).$$

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But, from above,

$$v = \sqrt{\left[ke^2/(mr_n)\right]} = n\hbar/(mr_n) \quad \text{or}$$

$$r_n = n^2\hbar^2/\left(kme^2\right).$$

The lowest allowed orbit has $r_1 = \hbar^2/(kme^2) = 0.53 \times 10^{-10}$ m and is called the Bohr radius. Next, we can write the allowed energies of the atom,

$$E_n = -mk^2e^4/\left(2n^2\hbar^2\right) \equiv -R/n^2.$$

R is called the Rydberg constant and has a value of 2.18×10^{-18} J $= 13.6$ eV. Now, the atom will emit a photon when an electron in level n 'jumps' to some lower level n'. A photon of energy $E_{nn'}$ will be emitted

$$E_{nn'} = R/n'^2 - R/n^2 = hf_{nn'} = hc/\lambda_{nn'}$$

$$1/\lambda_{nn'} = (R/hc)\left[1/n'^2 - 1/n^2\right] = (R/hc)\left[\left(n^2 - n'^2\right)/n^2n'^2\right]$$

$$\lambda_{nn'} = \left(n^2n'^2hc\right)/\left[R\left[\left(n^2 - n'^2\right)\right]\right] \quad \text{with } n > n'.$$

Now, if $n' = 2$,

$$\lambda_{n2} = (4n^2hc/R)/(n^2 - 2^2) = Bn^2/(n^2 - 2^2) \quad \text{where } B = 364 \text{ nm}.$$

Bohr's triumph is that this is exactly the Balmer formula for the wavelengths of the visible lines in the hydrogen spectrum!

Appendix G

Detecting spin

Suppose that you have a tiny bar magnet and put it in a uniform magnetic field; what will happen? As you can see in figure G.1, the north pole will be attracted to the south pole of the external magnet and the south pole will be attracted to the north pole with a force of equal magnitude but opposite direction. Therefore, the net force on the bar magnet is zero although there is a net torque tending to line the magnet up with the uniform field. So, if the bar magnet were shot through the field it would go through undeflected.

Now, suppose that the field were nonuniform with the field being stronger near the south pole of the external magnet than near the north pole. This could be achieved by shaping the pole pieces of the external magnet as shown in figure G.2. If the bar magnet happened to have its south pole oriented upwards as in the figure, there would be a net force on the bar magnet which was down as shown. If the north pole were oriented upwards, there would be a net force upwards. If the bar magnet were oriented horizontally, there would be no net force on it. So, if a bar magnet were shot through this field (imagine you're shooting it straight down into the page in figure G.2), its straight-line path would be deflected depending on its initial orientation. A collection of randomly oriented little bar magnets would result in a vertical smear of emerging trajectories.

An experiment was performed by the German physicists Otto Stern and Walther Gerlach in 1922 which attempted to observe the effect. A sketch of their experiment is shown in figure G.3. A small current loop has a magnetic field just the same as a small bar magnet. Therefore, with the Bohr model of orbiting electrons, the picture of the hydrogen atom in 1922, should behave like a tiny bar magnet and be deflected depending on its orientation when it entered the magnet. So it was expected that one would see a whole continuum of vertical deflections from the north-up to the north-down orientations. Instead what was seen was only two deflections, one up and one down. In terms of the Bohr model, this was not understandable.

doi:10.1088/978-1-6270-5939-8ch12

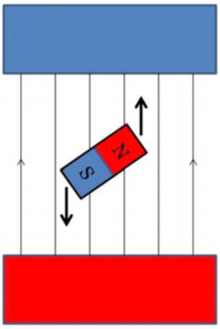

Figure G.1. Dipole in a uniform field.

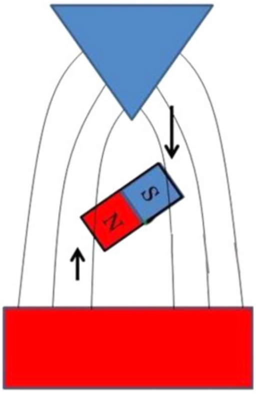

Figure G.2. Dipole in a non-uniform field.

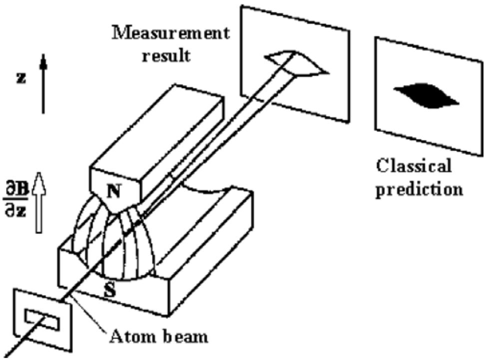

Figure G.3. The Stern–Gerlach experiment.

Was it understandable in terms of the Schrödinger equation solutions of the hydrogen model? At first blush, the answer seems to be yes because the quantum number m specifies the allowed orientations of the angular momentum, so the orbit could not take on any orientation relative to the vertical axis, only those corresponding to allowed m values. But, the allowed values of m run from $-\ell$ to ℓ and there would always be an odd number of them because ℓ is always an integer. For example, for $\ell = 2$, $m_\ell = -2, -1, 0, 1, 2$, a total of five. So, the experiment is a puzzle.

The results indicated that there were two values of m and so this would imply that $m = -\frac{1}{2}, +\frac{1}{2}$ which would imply that $\ell = \frac{1}{2}$. But we know this cannot be true. This is where it is realized that the electron must have intrinsic angular momentum and it must have a quantum number $s = \frac{1}{2}$. Its magnetic quantum number should be labeled m_s to avoid confusion with the magnetic quantum m_ℓ number associated with orbital angular momentum. The possible m_s states, $\pm\frac{1}{2}$, are often referred to as 'spin up' and 'spin down'. There is a wonderful animation of the Stern–Gerlach experiment on Wikipedia.

Finally, it should be noted that what is really being observed is the magnetic dipole moment of the electron, not the spin. A small magnet with N and S poles is called a magnetic dipole; the *moment* is a measure of the strength of the magnet. The fact that the deflections are the way they are lets us infer that the spin angular momentum quantum number must be $\frac{1}{2}$ and that there is an angular momentum associated with the magnetic moment.

Appendix H

Chronology

1896–98: Henri Becquerel, and Marie and Pierre Curie discover radioactivity.
1897: J J Thompson discovers the electron.
1899–1900: Ernest Rutherford and others separate radioactivity into α- and β-radiation.
1900: Villard identifies a third component of radioactivity, γ-radiation.
1900: Max Planck explains black-body radiation.
1905: Albert Einstein explains the photoelectric effect, introduces the photon.
1907: Rutherford determines that an α-particle is the nucleus of ^{4}He.
1908–13: Rutherford scattering, nuclear model.
1909: Milliken measures the electron charge.
1913: Niels Bohr proposes his model of the hydrogen atom.
1922: Stern–Gerlach experiment, electron spin.
1923: Compton scattering experiment.
1924: Louis de Broglie hypothesis, particle waves.
1924: Wolfgang Pauli proposes a new ‘quantum degree of freedom’ with two values.
1925: Schrödinger equation.
1925: Uhlenbeck and Goudsmit interpret Pauli’s degree of freedom as an angular momentum, spin.
1925: Pauli exclusion principle.
1928: George Gamow explains α-decay.
1930: Wolfgang Pauli proposes the neutrino to preserve energy conservation in β-decay but calls them neutrons.
1932: Chadwick discovers the neutron.
1933: Enrico Fermi suggests renaming Pauli’s neutron to neutrino to avoid ambiguity with the recently discovered neutron.
1932: Anderson discovers positrons (β^+) in cosmic rays.
1934: Frederic and Irene Joliot-Curie discover β^+-decay.
1934: Neutron-induced fission of uranium discovered Fermi and Otto Hahn.

doi:10.1088/978-1-6270-5939-8ch13

1937: Alvarez discovers electron capture.
1940: Spontaneous fission discovered by Flerov and Petrzhak.
1942: First nuclear reactor in Chicago by Fermi and colleagues.
1956: Cowan and Reines experimentally observe neutrinos.

Appendix I

Miscellaneous calculations and computations

I.1 Electrons in the nucleus

We learned in chapter 4 that one of the earliest models of the nucleus was that there were A protons and $A–Z$ electrons making the total charge of the nucleus Ze as had to be the case if there were Z electrons outside the nucleus. One of the most serious objections to this model had to do with the problems encountered by trying to confine the electron to such a small space. The diameter of a medium-sized nucleus is about

$$d \sim 10^{-14}\ \text{m} = 10\ \text{fm}.$$

If we think of the electron as having a wave function which is confined to this length and is zero at the edges of the nucleus then the longest wavelength λ (corresponding to the smallest possible momentum p) which would fit would have

$$\lambda \sim 2d = 20\ \text{fm}.$$

Using the de Broglie hypothesis to relate λ and p,

$$\lambda = h/p = 2d, \quad \text{so}$$
$$p = h/2d \approx \left(4 \times 10^{-21}\ \text{MeV}\cdot\text{s}\right)/(20\ \text{fm}) = 2 \times 10^{-22}\ \text{MeV}\cdot\text{s fm}^{-1}.$$

Now, it was shown in appendix B that the total energy of a particle of mass m and momentum p is

$$E = \sqrt{\left(p^2c^2 + m^2c^4\right)},$$

so we need to calculate pc. The speed of light is

$$c = 3 \times 10^8\left(\text{m s}^{-1}\right) \times \left(10^{15}\ \text{fm}/1\ \text{m}\right) = 3 \times 10^{23}\ \text{fm s}^{-1}.$$

doi:10.1088/978-1-6270-5939-8ch14

Finally,

$$pc \sim 60 \text{ MeV}.$$

The rest mass of an electron is about

$$mc^2 = 0.5 \text{ MeV},$$

so the total energy of the electron is

$$E \sim \sqrt{\left(p^2c^2 + m^2c^4\right)} = \sqrt{\left(60^2 + 0.5^2\right)} \approx 60 \text{ MeV}.$$

The kinetic energy of a particle is its total energy minus its rest-mass energy, and the rest-mass energy is negligibly small compared to *E*. Having electrons of this energy was totally inconsistent with what was known about nuclei in the 1920s. All energies are on the order a few MeV, not tens of MeV.

I.2 Electrons in the atom

It is also instructive to perform the same calculation for an electron in the atom where

$$d \sim 10^{-10} \text{ m} = 0.1 \text{ nm}$$

So, following what was done above,

$$\lambda \sim 2\text{d} = 0.2 \text{ nm}.$$

Using the de Broglie hypothesis to relate λ and p,

$$\lambda = h/p = 2d, \quad \text{so} \ \ p = h/2d \approx \left(4 \times 10^{-15} \text{ eV} \cdot \text{s}\right)/(0.2 \text{ nm}) = 2 \times 10^{-14} \text{ eV} \cdot \text{s nm}^{-1}.$$

Now, it was shown in appendix B that the total energy of a particle of mass m and momentum p is

$$E = \sqrt{\left(p^2c^2 + m^2c^4\right)},$$

so we need to calculate pc. The speed of light is

$$c = 3 \times 10^8\left(\text{m s}^{-1}\right) \times \left(10^9 \text{ nm}/1 \text{ m}\right) = 3 \times 10^{17} \text{ nm s}^{-1}.$$

Finally,

$$pc \sim \left(2 \times 10^{-14}\right)\left(3 \times 10^{17}\right) = 6 \times 10^3 \text{ eV}.$$

The rest mass of an electron is about

$$mc^2 = 0.5 \text{ MeV} = 5 \times 10^5 \text{ eV},$$

so the total energy of the electron is

$$E \sim \sqrt{\left(p^2c^2 + m^2c^4\right)} = \sqrt{\left(\left(6 \times 10^3\right)^2 + \left(5 \times 10^5\right)^2\right)}.$$

Since the energy is seen to be almost all rest-mass energy, it is convenient to calculate the kinetic energy by expanding E,

$$K = E - mc^2 = mc^2\left(1 + \left(p^2c^2/m^2c^4\right)\right)^{1/2} - mc^2$$
$$\approx mc^2\left(1 + \tfrac{1}{2}\left(p^2c^2/m^2c^4\right) + \cdots\right) - mc^2 = \tfrac{1}{2}\left(p^2c^2/mc^2\right) + \cdots = 36\ \text{eV}.$$

This is the correct order of magnitude of typical electron energies in atoms as was shown in chapter 3.

I.3 Photon–proton backscattering

In section 4.2. a 'mystery radiation' was found to backscatter (180° scattering) from protons at rest and the protons had a final kinetic energy of about 5 MeV. It was asserted that if the 'mystery radiation' was photons, the incident photon energy would need to be more than 50 MeV. This will be demonstrated in this section. In the following, I will approximate $m_pc^2 \approx 1000$ MeV and $E'_p = m_pc^2 + K \approx 1005$ MeV. The incident (final) photon momentum will be denoted $p_\gamma\,(p'_\gamma)$; the initial (final) proton momentum will be denoted $P = 0\ (P')$; the initial (final) photon energy is $E_\gamma = p_\gamma c\,(E'_\gamma = p'_\gamma c)$. Conserve energy:

$$p_\gamma c + 1000 = p'_\gamma c + 1005$$
$$p_\gamma c = p'_\gamma c + 5.$$

Conserve momentum:

$$p_\gamma = -p'_\gamma + P'.$$

Note the negative sign which is because the momentum reverses its direction. Therefore, p'_γ is the magnitude of the vector and should therefore be a positive number when we solve for it. Next multiply by c,

$$p_\gamma c = -p'_\gamma c + P'c.$$

Add this to the energy conservation equation:

$$2p_\gamma c = P'c + 5.$$

Evaluate $P'c$:

$$E'_p = 1005 = \sqrt{\left(P'^2c^2 + 1000^2\right)}$$
$$P'c = 100\ \text{MeV}.$$

So,

$$p_\gamma c = E_\gamma = 52.5\ \text{MeV}$$

$$p'_\gamma c = E'_\gamma = (52.5 - 5) = 47.5\ \text{MeV}.$$

One thing to notice here is that the physics is no different from the Compton scattering discussed in appendix C, a relativistic elastic scattering calculation.

I.4 Rate of spontaneous fission in ^{235}U

The relation between the decay constant λ and half-life $T_{½}$ is

$$\lambda = \ln(2)/T_{½} = 0.692/T_{½},$$

so, if

$$T_{½} \approx 3.5 \times 10^{17}\ \mathrm{yr}$$

for spontaneous fission (SF), then

$$\lambda \approx 2 \times 10^{-18}\ \mathrm{yr}^{-1}.$$

A mole of ^{235}U is 235 g and contains about 6×10^{23} atoms, so the number of atoms in one gram is about

$$N = 6 \times 10^{23}/235 = 2.6 \times 10^{21}.$$

So,

$$|\mathrm{d}N/\mathrm{d}t| = \lambda N \approx 5000\ \mathrm{SF\ g^{-1}\,yr^{-1}} \approx 14\ \mathrm{SF\ g^{-1}\,d^{-1}}.$$