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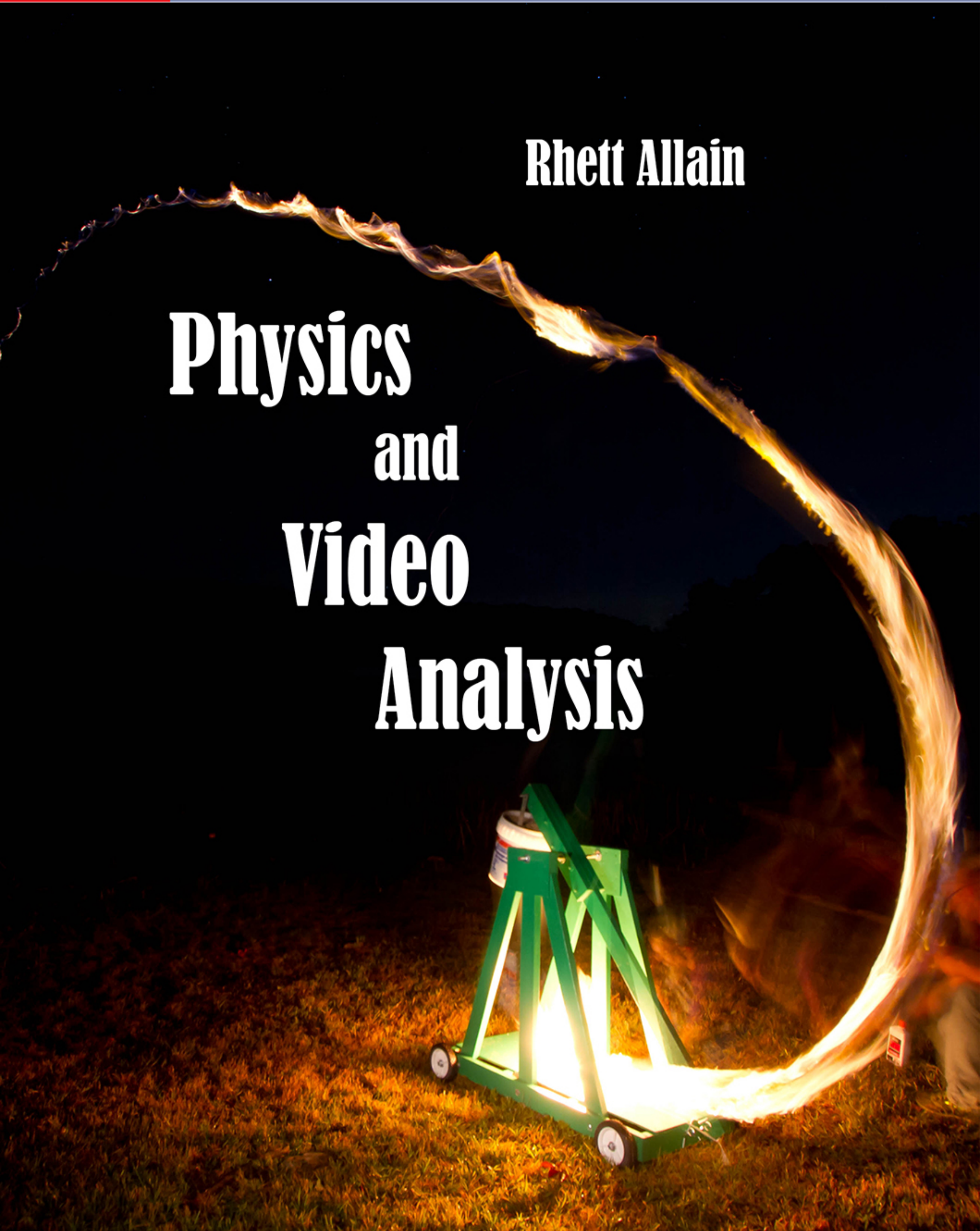
Rhett Allain

Physics and Video Analysis



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COVER ART
A flaming gourd launched by The Free Energy Flyer at the 26th Annual Gibbs Conference on Biothermodynamics, Carbondale Ill. No animals or small children were harmed before, during or after launch. Used by permission of the photographers and organizers, Drs. Lance Hellman and Brian Baker, University of Notre Dame, Department of Chemistry and Biochemistry.

This book is dedicated to every person that has shared an online video. Without your collective diligence in posting or sharing cat videos and other crazy videos, I would have very little video analysis material to work with.

I would like to also dedicate this to my wife Ashley and my family for their continued support in things that at some point seem trivial. This is especially true for my children for the countless times they have been called into the backyard to record yet another silly video.

Contents

Preface

Of course I had been aware of the idea of video analysis for quite some time. However, it was not until I saw a particular video online that showed Kobe Bryant apparently jumping over a pool of snakes that I started really using video analysis. It still makes me excited to take a video and squeeze hidden secrets from its individual frames looking for a sign that it could be fake.

To this day, I still use video analysis for many posts on my blog, Dot Physics, at Wired Science Blogs (http://www.wired.com/wiredscience/dotphysics). It is this passion for video analysis that is the basis for this book. Here I have combined the basic techniques of video analysis with some of my favorite examples.

Acknowledgements

I would like to thank Doug Brown, the creator of my favorite video analysis software—Tracker Video Analysis for his contributions to the field of video analysis as well as producing an excellent and free piece of software.

Author biography

Rhett Allain

Rhett Allain is currently an Associate Professor of Physics at Southeastern Louisiana University and blogger at Wired Science Blogs. Early in high school, he realized that he had an interest in physics and went on to Benedictine University where he earned a BSc in physics. After that he graduated from the University of Alabama with an MSc in physics and North Carolina State University with a PhD in physics specializing in physics education research.

During graduate school, Allain realized that he enjoyed teaching physics because only by teaching do you really start to learn physics. In 2008 he started to write blog posts analyzing fun things in physics as a way to show students how to complete small research projects. After just a couple of posts, he became addicted to blogging and has continued writing about things that do not matter ever since. His favorite topics are estimations and the physics of science fiction.

Other than blogging, Rhett is also the author of the National Geographic book *Angry Birds Furious Forces* and *Geek Physics: Surprising Answers to the Planet's Most Interesting Questions* from Turner Publishing. He was also a science consultant for the National Geographic show *The Science of Stupid* and an occasional advisor for *The MythBusters*. Rhett also enjoys taking things apart although he usually cannot put them back together.

Chapter 1

Introduction to video analysis

The basic idea of video analysis is not that new. In fact, one of the earliest examples was a photographic experiment set up in 1878 [1]. The experiment was designed to address a question about the motion of a galloping horse. Do horses ever lift all four feet off the ground during their gallop? It is actually difficult to see with the unaided eye, since the process is quite fast (as fast as a galloping horse). The experiment used multiple cameras to obtain 24 still images at different times during the horse's movement.

If you put all these photographs together, you get the short video shown in figure 1.1.

Figure 1.1. Animated gif of a horse running. Reproduced from https://en.wikipedia.org/wiki/Sallie_Gardner_at_a_Gallop.

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Although this series of images was just used to look at the position of the horse at different times, it is not all that different from modern video analysis. In short, the goal of video analysis is to obtain position and time data from the frames of a video.

Think of the most basic example—a video of a ball tossed into the air. If the video is recorded at 30 frames per second, then each frame of the video lasts 1/30 of a second. Moving forward by one frame is the same as moving 1/30th of a second forward. Now suppose the video is also recorded at a resolution of 640×360 pixels. In each frame of the video, I could determine the horizontal and vertical pixel location of the ball. If there is some object in the video of known size (such as a random meter stick laying around) then I can obtain a conversion between the pixel location and position in meters.

Putting this all together, I could create a plot of the horizontal and vertical motion as functions of time. Of course this is not the only thing that you can do with video analysis, but it demonstrates the power. But why even use video analysis at all? Why not just use a stopwatch and a meter stick to plot the motion of the ball? Or perhaps you could use a sound-based motion sensor to measure position at time. The answer is that often the horizontal (x) and vertical (y) motion is difficult to measure at the same time. In other cases, you might not have access to the actual experiment, but only to a video of the event. This is typically the case when examining interesting videos online or for examining the motions of events in cinematic movies.

By using video analysis, we are able to start with just a video and then measure the motions of an object or objects. It allows us to explore the world of physics using only videos that have already been created.

If you want to start using video analysis, there are natural questions to ask. Where do you go? What do you use? There are countless options. Let me list a few of the video analysis software options that exist today. These are in no particular order.

- **Tracker Video Analysis** [2]. This is a free video analysis package that also includes modeling tools. It is a Java program built on the Open Source Physics platform that runs on Mac OS X, Windows and Linux. Tracker Video includes many elements, such as video filters, perspective correction and autotracking. Since it is free and runs on multiple platforms, all the examples in this book will be via Tracker Video, although most of the ideas can be applied to other software packages.
- **Vernier's Logger Pro** [3]. This commercial software runs on both Mac OS X and Windows. It is the basic software used by Vernier to collect data from the Vernier sensors, but it also contains a video analysis element. Many students and faculty members find that Logger Pro is the ideal video analysis software because students might already be familiar with its graphing tools. Vernier also has a version of video analysis that runs on the iPhone.
- **Direct Measurement Videos (DMV)** [4]. This is not actually a software package. DMV are specially created videos that have annotations added so that users can easily make position and time measurements straight from the video. There is a DMV video player online that allows one to step frame by frame through the video. The main point of DMV is to let students participate

in video analysis without getting too involved in the tedious aspects of the software. DMV are freely available online.
- **Older software**. There are quite a few older software packages that can still be used but are no longer updated. VideoPoint and VideoGraph were two popular packages, but at this point it seems that neither is up to date.
- **No software**. Finally, it should be pointed out that you do not need any software other than a video player to perform video analysis. It is indeed possible to step through a video frame by frame to determine the times and you can use a ruler on the screen to measure positions.

These are just the most popular software platforms. My advice is to pick one and then become very familiar with your chosen tool. Once you have mastered your software you can begin to explore your favorite videos.

References

[1] https://en.wikipedia.org/wiki/Sallie_Gardner_at_a_Gallop
[2] http://physlets.org/tracker/
[3] www.vernier.com/products/software/lp
[4] https://serc.carleton.edu/sp/library/dmvideos/index.html

Rhett Allain

Chapter 2

Choosing and finding appropriate videos for analysis

Now you are ready to analyze some videos. Where do you start? The first step is to get a video. There are two things you can do. First, you can record your own. Second, you can find a video from somewhere (probably online). Before going over some of the details for both making and finding videos, let us look at what aspects of a video make it appropriate for video analysis.

Suppose you were to give every video you see a score. Perhaps we could call this the video analysis score (VAS). Videos with a higher VAS are easier and better to analyze. To obtain the VAS, you would add one point for each of the following criteria that the video meets.

Stationary camera and background. When you use a tripod with a camera, the background appears stationary. With a stationary background, an object's motion can be determined by measuring its position relative to this background. This means that you can fix your origin on some point in the background and it will be at rest relative to the ground. Consider the opposite. Imagine holding a camera with a shaky hand. In the video, the object of interest will have an apparent downward motion when the camera moves up. This means that to obtain the position of the object you would have to account for the apparent motion of the background. It is not impossible to do this, but it does involve extra work.

Camera does not zoom. A camera that zooms in or out is very similar to a moving camera. When a camera zooms in while an object is moving, the apparent size of everything also changes. In order to compensate for a zooming camera view you would need to measure the background scale in each frame. Again, this is not impossible, but it does mean a more difficult video to analyze.

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Object's motion is perpendicular to the camera. Suppose you take a ball and toss it towards a stationary camera. As the ball comes closer to the camera, its apparent size also becomes larger. This means that if you recorded the vertical position of this tossed ball, the apparent motion would appear greater in later video frames. Clearly this is something you would like to avoid. The best camera views have the camera pointing perpendicularly to the motion of the object.

Object's range of motion is small compared to the camera view. This is similar to the 'object perpendicular to the camera' item. If an object is moving perpendicularly to the view of the camera, but moves over a very large angle, then in fact it is also moving towards and then away from the camera. However, if the object is 10 m away from the camera and only moves 1 m perpendicularly to the camera, this effect is negligible.

Object is visible in every frame of the video. If you cannot see an object in a video, you cannot determine its position directly. This usually happens in a video when an object moves behind some other object for a short time interval. It is not a very large problem if you can see the object again a little later. It just means that you will have incomplete data.

The video does not have repeating frames. Older online videos often have this problem. When a video is recorded in one video codec and then converted to another format with a different frame rate, you can get repeated frames. Typically, when this repeating frame problem occurs the frames only repeat every 10 or so instances. When you have a frame followed by an identical frame, the position of the object remains the same for both frames, although the reported time (based on the frame) changes. This could give inaccurate data—it is particularly bad when using position data to calculate velocities.

Deinterlaced video. The video is recorded by scanning horizontal lines of an image. A video with a resolution of 480×620 has 480 of these horizontal lines. Older video cameras would first record the odd number lines (1, 3, 5, 7…) and then record the even numbered lines (0, 2, 4, 6…). When the video is played back, it mostly looks fine. However, if you stop the video and look at one frame, you might be able to tell that the horizontal lines are from different times. Modern cameras usually record all lines at the same time (progressive scan), such that this is no longer a problem.

Slow shutter speed. For each frame of a video, the camera has to collect light in order to make an image. It is possible that this image collection time is long enough to see significant movement of an object during that time. In this case you would see an object as a blur since the object would be in more than one location during the imaging process. This problem is a result of low shutter speeds. Figure 2.1 shows an example, with a dropping basketball and a tennis ball.

Figure 2.1. A basketball and a tennis ball dropped at the same time. This image shows the effect of slow shutter speed on fast moving objects. Credit: Rhett Allain.

Object visible with respect to the background. What if you take an orange ball and throw it in front of an orange background? You might not be surprised to find that you cannot really see the ball very well. This might seem like an obvious case to avoid, but what about a video of a snowball in front of a field of snow? If you cannot see the object against the background, you cannot measure it.

Interesting physics content. There is one last thing to consider. Is there anything interesting to look at in the video? Without something to analyze, it just does not make a very nice video analysis project.

Now that you have an idea of all the things that make a video perfect (or not so perfect) for an analysis, you can pay attention to all those online videos in a different way. You do not just have to enjoy the video as it is presented, you can consider what you can do with it using video analysis.

Rhett Allain

Chapter 3

Video analysis and basic projectile motion

Perhaps it is best to start with something simple. A cart rolling at a constant speed and moving in a straight line is quite simple, but it is also slightly boring. Instead, let us look at a case of projectile motion.

From a physics perspective, projectile motion applies to any situation in which there is motion with constant acceleration. Physicists usually reserve projectile motion for cases in which the acceleration is due to gravitational force, but any constant acceleration would have the same mathematical model.

When looking at the video of an object's motion, we are really considering two things. What is the horizontal velocity? What is vertical acceleration? Before looking at a video, let us consider the equations of motion for an object with only gravitational force on it. Suppose the object is launched with an angle θ and has an initial velocity of v_0. This would be a representation of the forces on this projectile motion object as seen in figure 3.1.

If I call the horizontal direction the x direction and the vertical direction the y direction, then I can write the following two force equations (equation (3.1)).

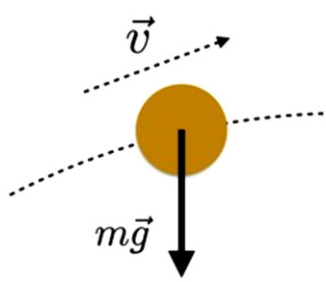

Figure 3.1. A diagram of a ball in motion under the influence of only the gravitational force.

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$$\begin{aligned} F_{\text{net-}x} &= ma_x = 0 \\ F_{\text{net-}y} &= ma_y = 0 \\ a_y &= -g. \end{aligned} \tag{3.1}$$

The g is the gravitational field near the surface of the Earth with a commonly accepted value of 9.8 N kg^{-1}. The initial x and y velocities can also be found as:

$$\begin{aligned} v_{x0} &= v_0 \cos\theta \\ v_{y0} &= v_0 \sin\theta. \end{aligned} \tag{3.2}$$

This assumes that the angle θ is measured from the horizontal axis. Since the x velocity is constant, the x position can be determined from the definition of the average velocity:

$$\begin{aligned} v_{\text{avg-}x} = v_{x0} &= \frac{\Delta x}{\Delta t} = \frac{x - x_0}{\Delta t} \\ x - x_0 &= v_{x0}\Delta t \\ x &= x_0 + v_{x0}\Delta t. \end{aligned} \tag{3.3}$$

Looking at the motion in the y direction:

$$\begin{aligned} v_{\text{avg-}y} &= \frac{v_y - v_{y0}}{2} = \frac{\Delta y}{\Delta t} = \frac{y - y_0}{\Delta t} \\ a = -g &= \frac{\Delta v_y}{\Delta t} = \frac{v_y - v_{y0}}{\Delta t} \\ v_y &= v_{y0} - g\Delta t \\ y &= y_0 + \left(\frac{v_y - v_{y0}}{2}\right)\Delta t. \end{aligned} \tag{3.4}$$

With some substitution, we obtain:

$$y = y_0 + v_{y0}\Delta t - \frac{1}{2}g(\Delta t)^2. \tag{3.5}$$

So, using the definition of acceleration in the x and y directions, we obtain two equations. In the x direction, the object should be moving with a constant velocity and in the y direction the object should have constant acceleration.

Now for a video analysis example using projectile motion. I am going to be using a video that is as simple as possible. This is a frame from the video showing a ball tossed in the air. For scale, there is a meter stick near one side of the frame, as seen in figure 3.2.

Next I will step through the process of analyzing this video. In this example, I am going to be using Tracker Video Analysis. Even though the screenshots are for a particular software package, the same idea will apply to other platforms.

Step 1: video setup. Once the video is loaded into the software, you need to set the scale. In this example I use the meter stick sitting on the right window (as shown).

Figure 3.2. A frame of a video showing the motion of a ball used in projectile motion video analysis. Credit: Rhett Allain.

Ideally, you would like to have something that covers the size of the video (which the meter stick does not). With a large object to set the scale, the small deviations in making the ends of the object should be negligible compared to the total length. This would increase the accuracy of the scale.

There is another consideration, the placement of the origin in the video. Usually the location of the origin is irrelevant. However, the orientation of the y axis should be in the vertical direction. In this video, the camera has a very slight tilt. By aligning the x axis along a row of bricks, we have a reasonable assumption that the y axis is vertical (figure 3.3).

Finally, there is one more thing to check. Most videos play at the same rate at which they have been recorded. However, a slow-motion video might be played back at a different speed. You want the real time between video frames, not the playback speed. This particular video was recorded at 240 frames s^{-1}. That means that the time interval from one frame to the next is 1/240 s.

Step 2: mark the location of the ball. The different video analysis software packages work slightly differently. However, most of them allow you to manually mark the location of the ball in each frame. During this process, the program will record the x, y position along with the time. For the example of the tossed ball, figure 3.4 gives a plot of the horizontal position versus time.

In figure 3.4 you can see that the data appear to be mostly linear. There seems to be a slight decrease in the horizontal velocity and this could be due to a small air resistance force on the ball. Still, the fit is good enough to serve as an example. By fitting a linear function to the data, we can see that the slope of this line is 6.6 m s^{-1} and this is the horizontal velocity of the ball.

Now, moving to the vertical position of the ball, see figure 3.5. Fitting a parabola to the data in figure 3.5, the term in front of the t^2 is -4.859 m s^{-2}. If we compare this

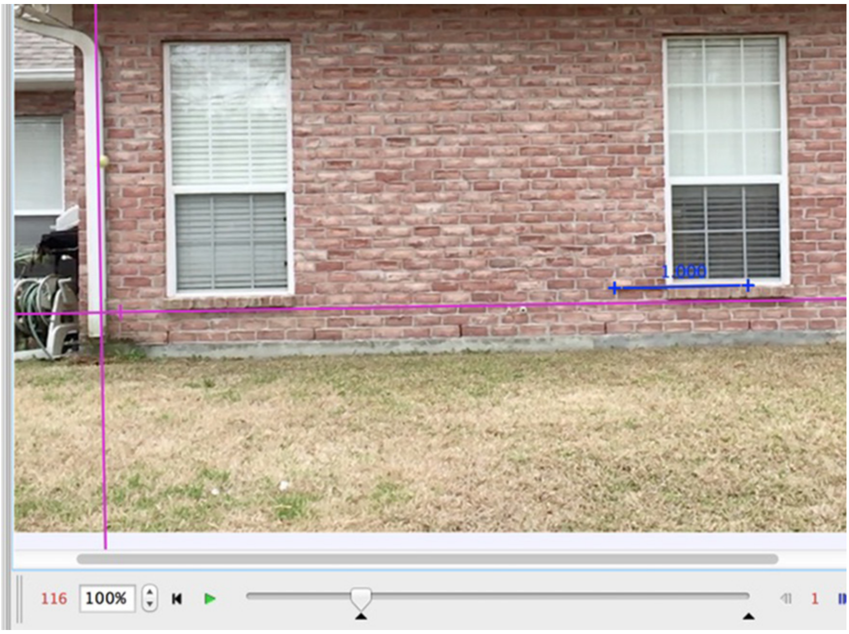

Figure 3.3. Screen shot from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the coordinate system and the scale. Credit: Rhett Allain.

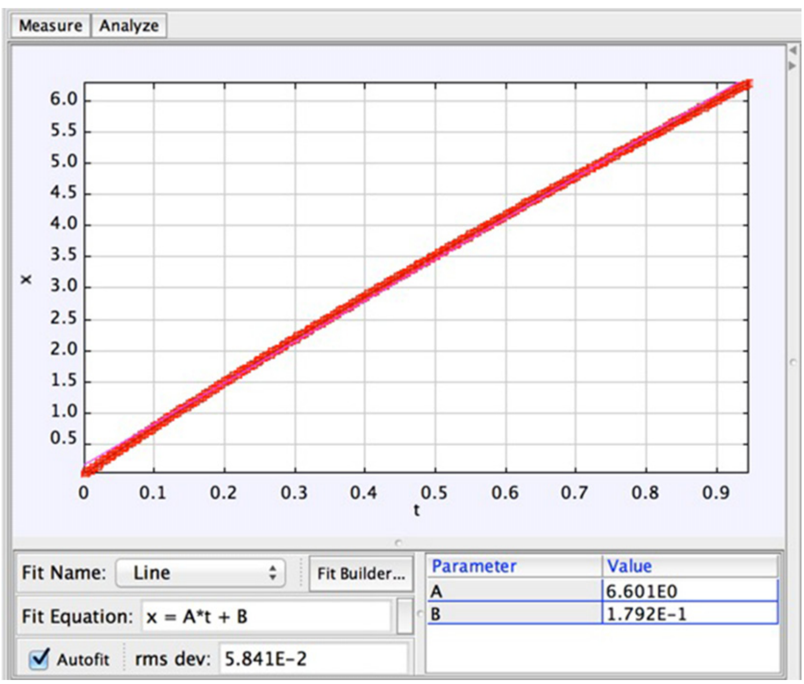

Figure 3.4. Screen shot from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the horizontal motion of the ball along with a linear fit.

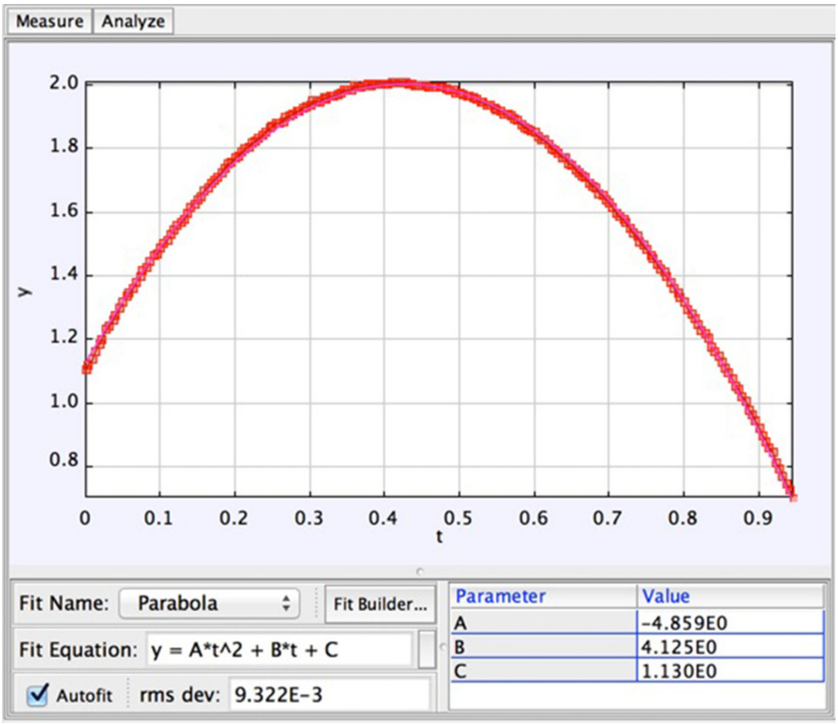

Figure 3.5. Screen shot from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the vertical motion of the ball along with a quadratic fit.

fit to the kinematic equation from above, we can see that the term in front of t^2 is (1/2)a. This means that the acceleration would be twice this coefficient at a value of -9.718 m s^{-2}. That is fairly close to the expected value of -9.8 m s^{-2}.

These are the two main things you would expect for projectile motion: a vertical acceleration of -9.8 m s^{-2} and a constant horizontal velocity. We can also find the initial velocity of the ball. Since we already know the horizontal velocity, we just need the initial vertical velocity. There are a couple of ways to obtain this initial velocity.

First, we can make a plot of the vertical velocity as a function of time. Most software packages will perform this calculation for you. Here is the plot from the ball toss video above (figure 3.6).

From this plot you can see that the slope also gives roughly the same acceleration (around -9.8 m s^{-2}). Even though the data looks a bit rougher, it is still fairly linear. The value of this function at time $t = 0$ s is 4.197 m s^{-1}. This is the initial vertical velocity.

The second method to determine the initial velocity is to use the parabolic fit of the y versus t data. From this, the term in front of the t value is the initial velocity. Looking back at the plot, this gives an initial velocity of 4.125 m s^{-1}.

There is one final method. If you look at the y versus t plot (figure 3.7), you can fit a linear function just to the very beginning of the data. The figure shows what that would look like.

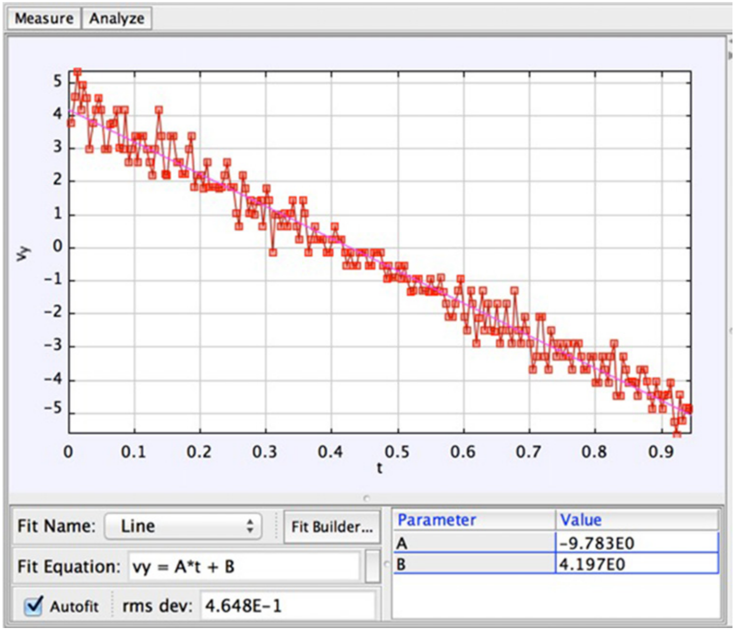

Figure 3.6. Screen shot from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the vertical velocity of the ball.

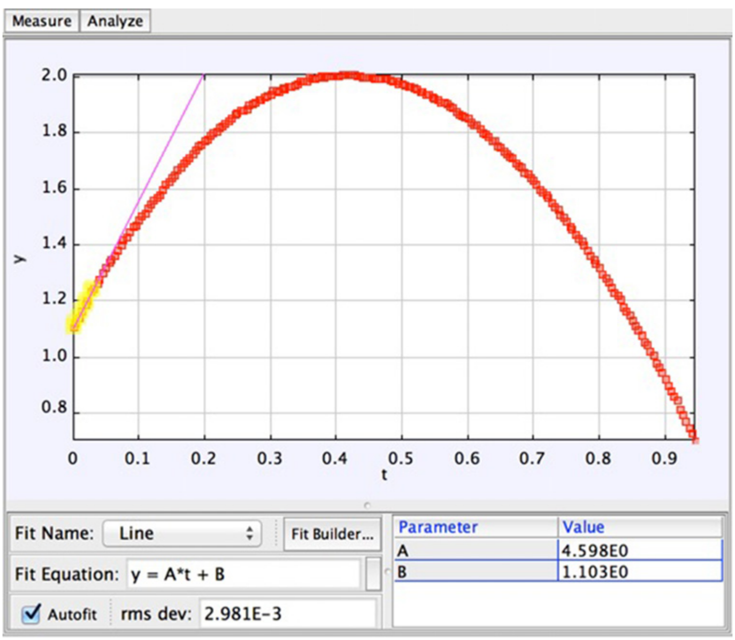

Figure 3.7. Screen shot from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the vertical position of the ball along with a linear fit at the beginning to get an estimate of the vertical velocity.

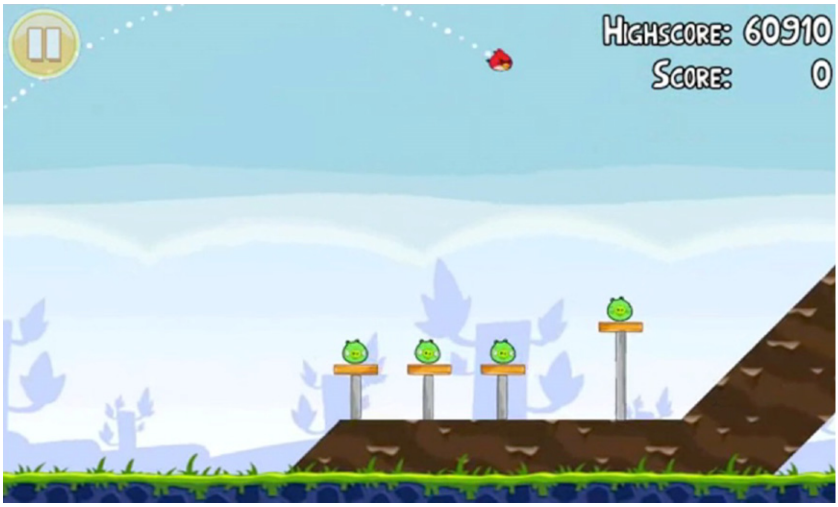

Figure 3.8. Screen capture from *Angry Birds*. Reproduced with permission from Rovio.

From the slope of this linear fit, we obtain an initial velocity of 4.598 m s^{-1}. Clearly this is a little bit different from the other two methods. It seems that the other methods are slightly more accurate because they do not have the assumption of a constant velocity over a short time interval.

So that is the first example of video analysis with a very simple case. Now we are ready to move on to more complicated things.

Let us start with an only slightly more complicated video showing projectile motion. This is a screenshot from the popular game *Angry Birds* (figure 3.8). The goal of the game is to launch a bird with a slingshot in the hope of destroying a pig. In order to make the game more playable, the scene automatically zooms and pans to focus on the motion of the bird.

Let me start with an analysis, just as I did in the previous example. I will scale the video and set the origin. In this case, I will call the length of the slingshot a distance of 1 unit (we do not have a real scale since it is just a video game). Figure 3.9 shows a plot of the bird's trajectory without changing the scale or origin.

Clearly this plot does not look like a parabola, as we would expect with projectile motion. Also, note that the bird left the frame of the video for several frames. If the video only had panning, we could manually fix the problem. I could mark the location of the origin in each frame of the video and then use that to adjust the measured position of the bird. I could also account for the changing scale by determining a scale factor for each frame by measuring the length of a known object.

Most video analysis programs have some method for dealing with these two problems. In fact, there can also be a third issue: rotating (tilting) video. The *Angry Birds* game does not tilt, but dealing with that is essentially the same as handling a moving camera.

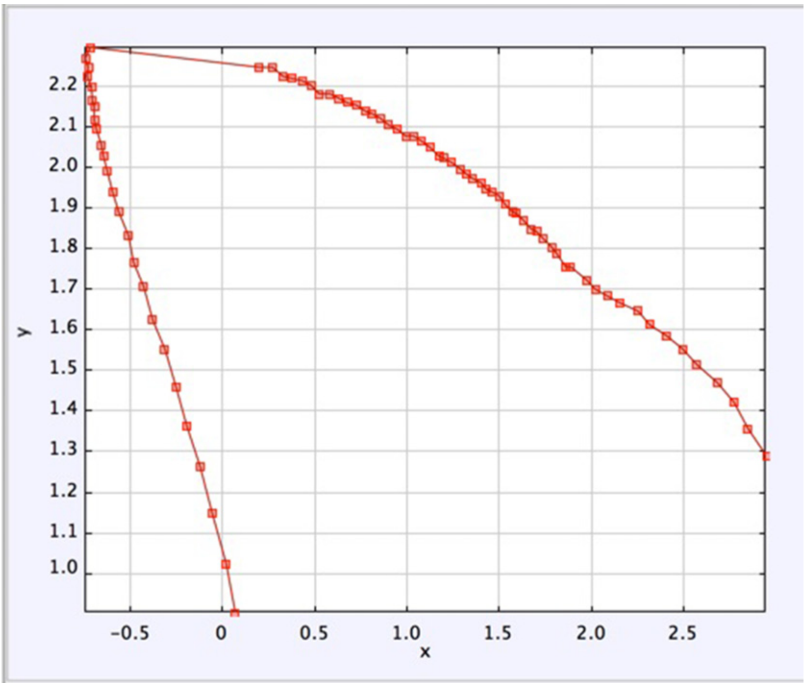

Figure 3.9. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing trajectory of a bird in Angry Birds without corrections for background motion.

But how do you handle these changing scenes in video analysis? Most software packages have a method for you to move the origin and change the scale in each frame. However, this video makes those methods challenging. If you want to move the origin, you need to pick some place to put the origin that you can see in every frame. There is no such point in this video. If you choose to simply move the location of the origin you would have to move it off the frame of the video for some instances. The same is true for changing the scale in each frame. You need to find some object you can see in every frame. The only object you can see in most frames is the bird itself. Using the bird for a scale is not the wisest decision. Since it is relatively small, a small error in the scale could mean a large error in the trajectory.

The Tracker Video Analysis software has a unique solution to this scaling–moving frame problem. It is called 'calibration points'. The idea is to use two points on the background of the video to determine both the change in scale and the motion of the background. Since you can add multiple sets of calibration points as the old ones move out of view, you can add two new ones.

By changing the scale and the location of the origin, I obtain the plot of the horizontal position of the bird shown in figure 3.10.

Remember that the bird goes out of view for part of the motion. However, the x motion still shows a constant x velocity, which is what we would expect for projectile motion. Now, looking at the vertical motion we obtain the plot in figure 3.11.

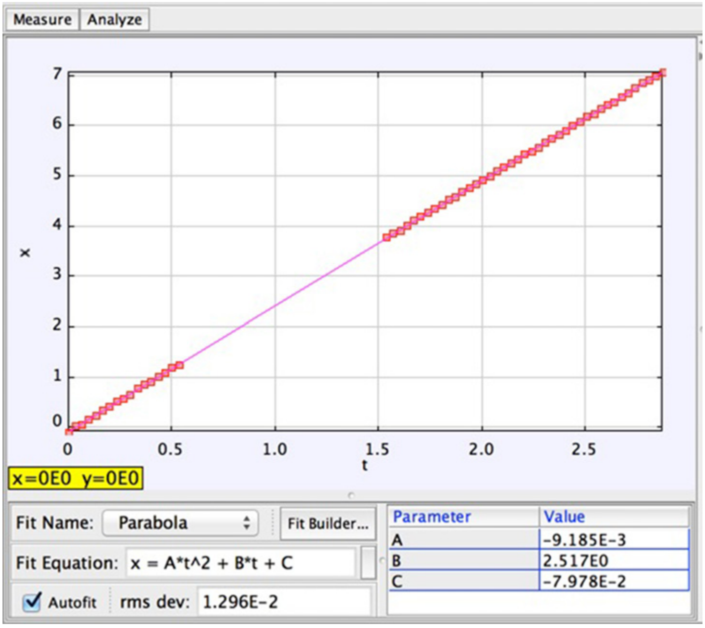

Figure 3.10. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the horizontal motion of a bird in Angry Birds after correction for scale and background motion.

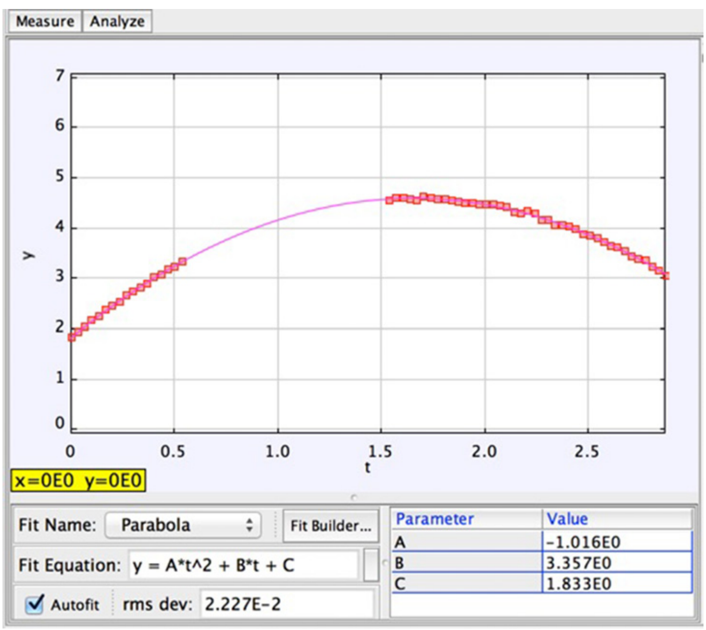

Figure 3.11. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the vertical motion of a bird in Angry Birds after correction for scale and background motion.

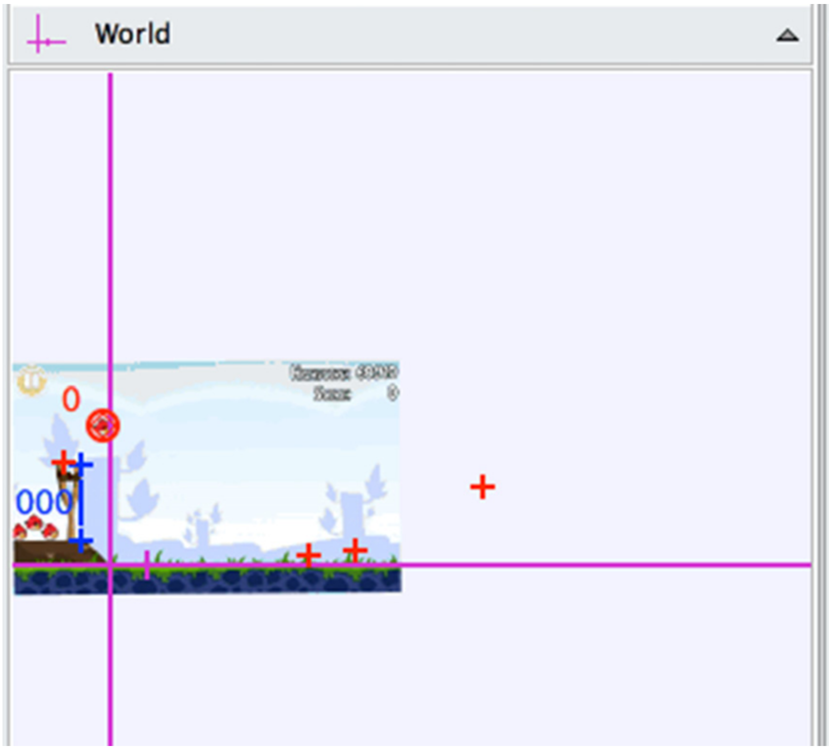

Figure 3.12. Animated view of Angry Birds removing background motion. This animation was created in Tracker Video Analysis. (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).)

Again, this is the curve that we would expect for projectile motion. Looking at the fit of the parabola, this gives a vertical acceleration of around −2 slingshots per second squared (not meters). What is the real acceleration in this video game? Well, since it is just a game you would have to make some assumptions about the size of things or the correct acceleration. It is possible to read more about scaling in the *Angry Birds* game[1].

There is one final thing: the output from Tracker Video Analysis that shows the view of the same scene without panning or zooming is given in figure 3.12.

That is it for basic projectile motion. I have covered all the technical details you would need in order to perform a complete analysis of a fairly simple video using Tracker Video Analysis. If you wanted to do this with a different software package, it would mostly be the same idea. It is also important to note that most of the videos you see would follow an analysis very similar to this. Of course, the more complicated videos are also more interesting. In the following chapters we see how to handle different types of videos.

[1] www.wired.com/2010/10/physics-of-angry-birds.

Rhett Allain

Chapter 4

Video analysis with perspective correction

In chapter 2, I stated that video analysis works best when the motion of an object is perpendicular to the view of the camera. Why is this better? Let us suppose that there was a ball that was tossed towards a camera. Figure 4.1 shows the path of motion (along with the camera) as viewed from above.

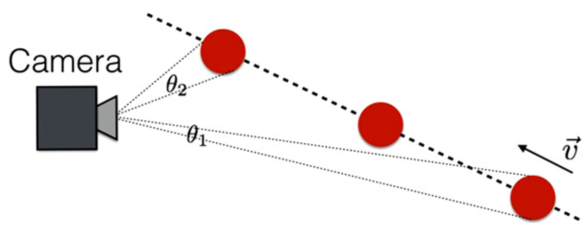

Figure 4.1. Diagram showing the angular size of an object as it moves towards a camera.

At first, the ball is further away from the camera. This means that there it will have a smaller angular size θ_1. Since video analysis uses angular size to determine size and scale, this object would appear to be smaller than it is in a later frame (with angular size θ_2). This means that if you used plain video analysis methods, the ball would appear to be moving faster the closer it gets to the camera. Of course this would not be true, but how do you fix it?

One method for dealing with non-perpendicular motion is to use perspective correction. I am not certain about all the software packages, but I do know that Tracker Video Analysis has a perspective correction tool. Instead of going through a full tutorial, let me just go over the basic principles of how it works.

doi:10.1088/978-1-6817-4067-6ch4

Let us start with the video in figure 4.2.

Figure 4.2. Animated gif based upon [1] showing the landing of a drone on an aircraft carrier.

This is a video from the US Navy showing the tests of carrier landings for a drone aircraft [1]. The drone comes onto the runway at an angle with respect to the camera. With Tracker Video Analysis, we can warp the video to make the motion appear perpendicular to the camera. To start, we need the actual dimensions of something in the background. Fortunately, we can use the size of the landing runway itself.

Figure 4.3 shows four points on the runway that should form a rectangle if viewed from above.

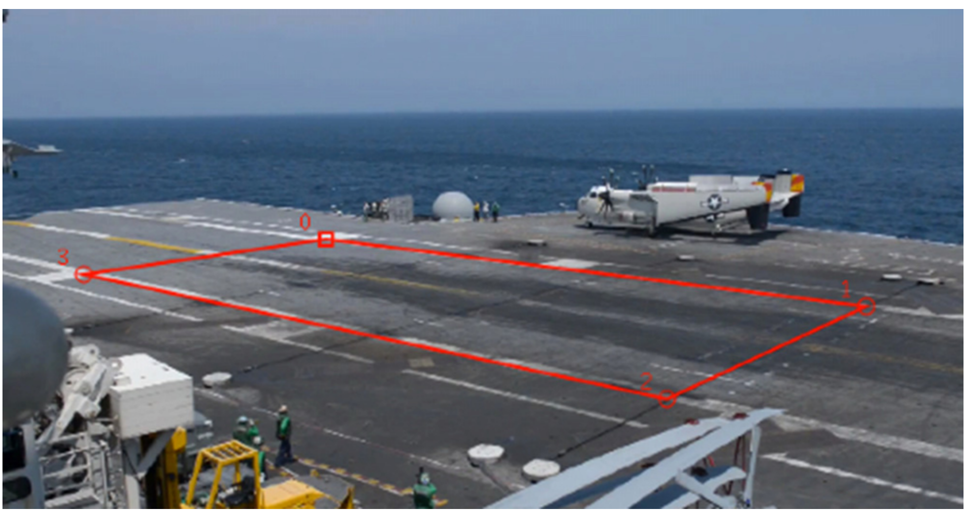

Figure 4.3. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the location of a rectangle used to correct for perspective. Based on video [1].

With a bit of research, I can find that the distance between the white lines on the landing strip is 26.7 m and the length of the landing for the corners is 53.4 m. After entering these values into Tracker Video, it produces figure 4.4.

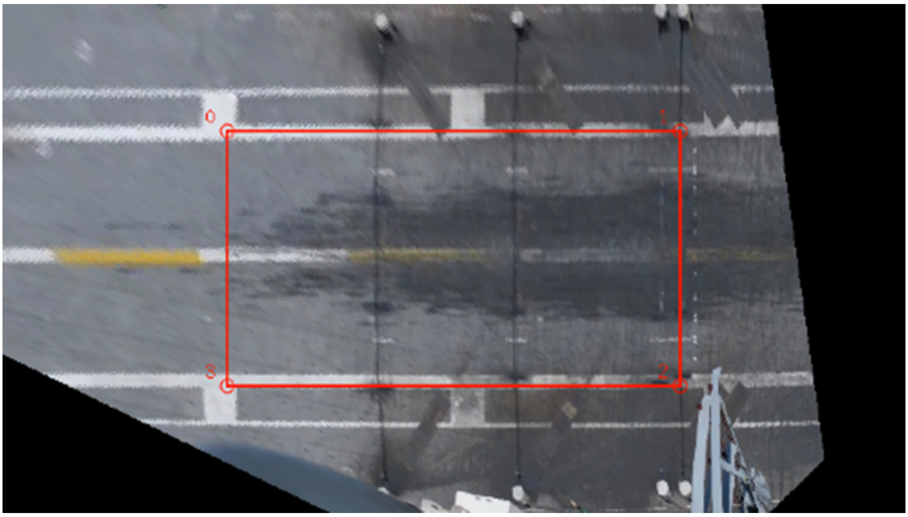

Figure 4.4. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing perspective corrected image. This is based upon [1].

In this corrected video, I can now mark the location of an object to obtain the position. For this case, the drone is actually above the runway for part of the flight. In order to use a point that is in the perspective corrected plane, I mark the location of the drone's shadow (figure 4.5).

Figure 4.5. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the marked locations of a drone as it lands. This is based upon [1].

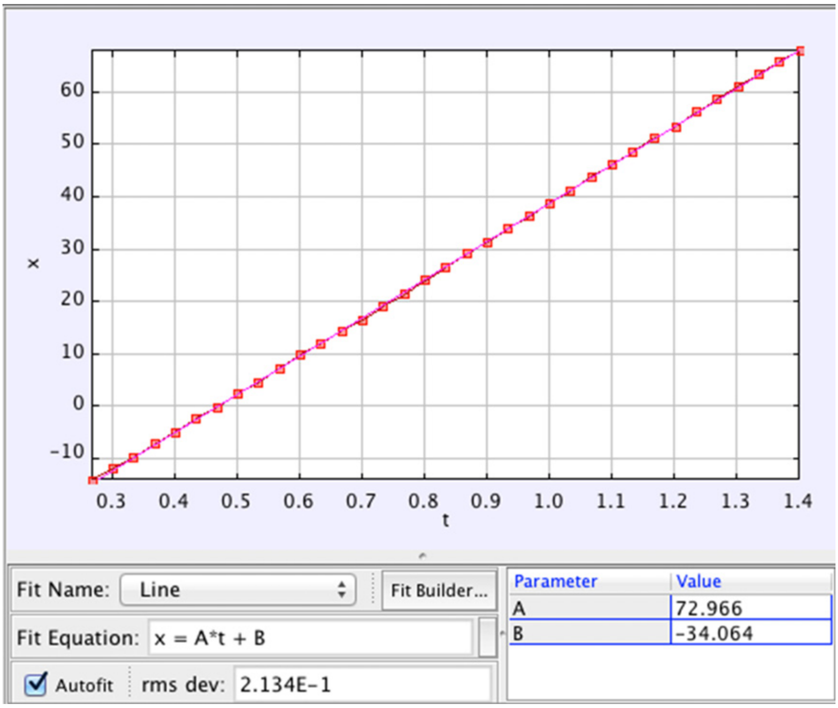

Figure 4.6. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the position of the landing drone in the perspective corrected coordinate system.

After marking all of these points, I obtain the plot of position versus time shown in figure 4.6.

From the slope of this line, the drone had a speed of around $72.97\,\mathrm{m\,s^{-1}}$ or 163 mph. That seems to be a reasonable speed. Also, the horizontal appears to be mostly constant. This is what you would expect for an aircraft during the landing process (but before catching the arrestor cable).

In this chapter, we have looked at a more difficult video analysis example. The motion was simpler than projectile motion since it was mostly in just one dimension. However, the video that we started with had some issues with perspective. Using some techniques to adjust the video, we can still obtain reasonable data for the motion of the object.

Reference

[1] www.youtube.com/watch?v=cPaH8CCtRVU

Rhett Allain

Chapter 5

Using angular size

What if an object is only moving towards or away from the camera? Can you still plot the position of this object? You cannot use perspective correction since there might not be a scale next to the object that can be corrected. You cannot use normal video analysis methods since the object is not moving perpendicularly to the camera.

There is one trick you can use. You can use the changing angular size of an object to obtain a value for the distance from that object to the camera. First, I will quickly review angular size.

Figure 5.1 shows an object of length L a distance r away from some camera (the circle is the camera). The further away from the camera the object is, the smaller the angular size of the object (theta). This is not anything new. You can see this for yourself. Close one eye and put your thumb out an arm's length from your head. How big does your thumb appear? Now move it closer to your eye: it appears to be larger.

We can obtain the following relationship between object size and angular size:

$$
\begin{aligned}
L &= r\theta \\
r &= \frac{L}{\theta}.
\end{aligned}
\tag{5.1}
$$

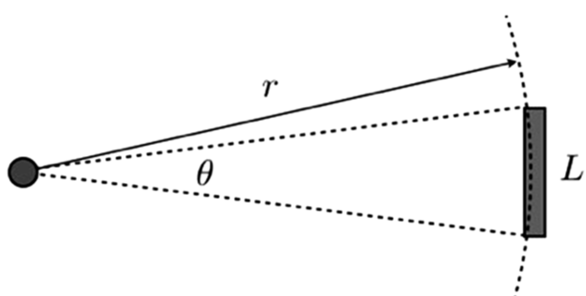

Figure 5.1. Diagram showing the relationship between distance, object size and angular size.

doi:10.1088/978-1-6817-4067-6ch5

Figure 5.2. Animated gif based upon [2] showing the motion of a balloon as seen from a camera mounted on the balloon. © Meryl McCurry. Obtained from Prezi website. SpaceCats clip reused under the Prezi Public User Content licence terms.

If you know the angular size of an object along with its actual size, you can find the distance to that object. If I measure the angular size for an object moving away from the camera, I can obtain position as a function of time. It seems simple, but of course there are always problems. Let us look at an example.

What if you put a camera on a weather balloon? This is exactly what John Burk did with his physics students [1]. Figure 5.2 shows a short clip of the view from the balloon as it ascended (in double speed so you can see the changes). The full video is available from [2].

Even if you put an altimeter in your balloon, could you also obtain a plot of the height as a function of time just from the video? Yes. I can measure the pixel sizes of objects on the ground to determine the height of the balloon. However, I need to know the angular size of each pixel. If you know the angular field of view for the camera, you can easily determine the angular size of objects in the frame. However, I do not know what kind of camera was used in this experiment. For example, the iPhone 4 camera has a horizontal field of view of around 56°. Could you not just email the student or instructor and ask for the type of camera? In this case, yes—that is likely possible. But there are countless other videos where you cannot obtain the camera information. There is another method for finding the angular field of view for the camera.

Figure 5.3 shows a frame from before the balloon launched, when it was still near the ground. By estimating the size of objects on the ground as well as distances from the camera to the object, I obtain a value for the angular field of view of the camera. Figure 5.4 shows a view from another camera that shows the approximate position of the camera during that same shot.

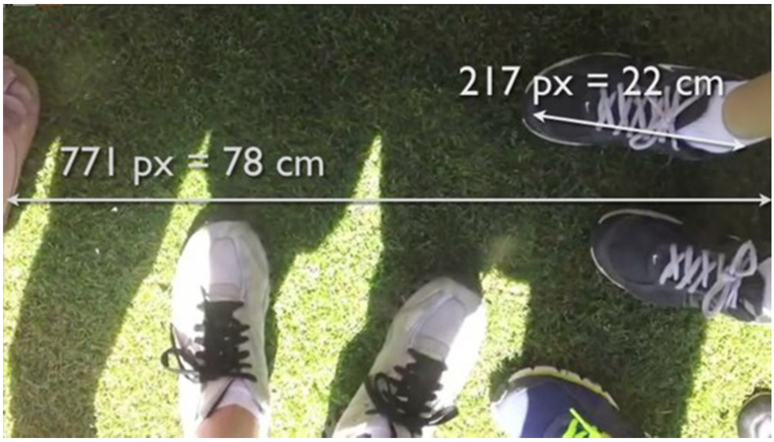

Figure 5.3. Estimated size of objects used to calculate the camera's angular field of view. This is based upon [2]. © Meryl McCurry. Obtained from Prezi website. SpaceCats still reused under the Prezi Public User Content licence terms.

Figure 5.4. View of balloon launch height also used to calculate the camera's angular field of view. This is based upon [2]. © Meryl McCurry. Obtained from Prezi website. SpaceCats still reused under the Prezi Public User Content licence terms.

From this, I obtain the following estimate for the angular field of view of the camera:

$$\theta_{\text{camera}} = \frac{L}{r} = \frac{78 \text{ cm}}{100 \text{ cm}} = 0.78 \text{ rad}. \tag{5.2}$$

This gives an angular field of view of 44.7°. That seems reasonable. Now that I know the angular field of view, I can obtain the angular size of any object in a frame by

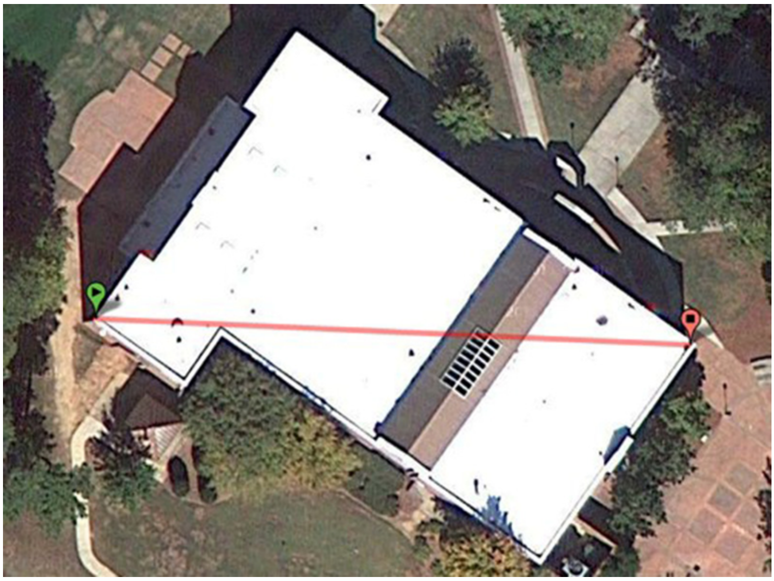

Figure 5.5. Screen capture from Google Maps. This image is used to measure the size of a building in the video. This is based upon [2].

setting the horizontal scale of the frame to 0.78 radians. After that all 'distances' will actually be in radians.

To start the video analysis of the balloon's motion, I now need an object with a known size. Figure 5.5 shows one of the buildings that you can see at the beginning of the video and this is the object I will use. The building can be seen shortly after the balloon lifts off.

According to Google Maps[1] the distance from one corner to the next is 67.5 m. So, I have the size of the object and I have the angular field of view of the camera. Of course the building will only work for so long. Eventually the balloon will get so high that the building appears too small to be useful. At that time, I can switch to a larger object—in this case the distance from the same building to the nearby baseball field. Figure 5.6 shows a plot of the angular size versus time for these two objects.

Now I can calculate the distance from each object to the camera based on the angular size. Figure 5.7 shows a plot of the height of the balloon (distance to the camera) as a function of time.

How fast was the balloon rising? The position versus time looks mostly linear during the first half and then the second half. Figure 5.8 shows the same data with two different linear function fits.

From the slopes of these lines, I obtain a rising speed of 3.2 $\mathrm{m\,s^{-1}}$ for the beginning and then 4.5 $\mathrm{m\,s^{-1}}$ for the higher altitudes.

So you can see that even in the case of an object moving towards or away from a camera (or in this case, the camera moving away from the ground), you can still

[1] http://maps.google.com.

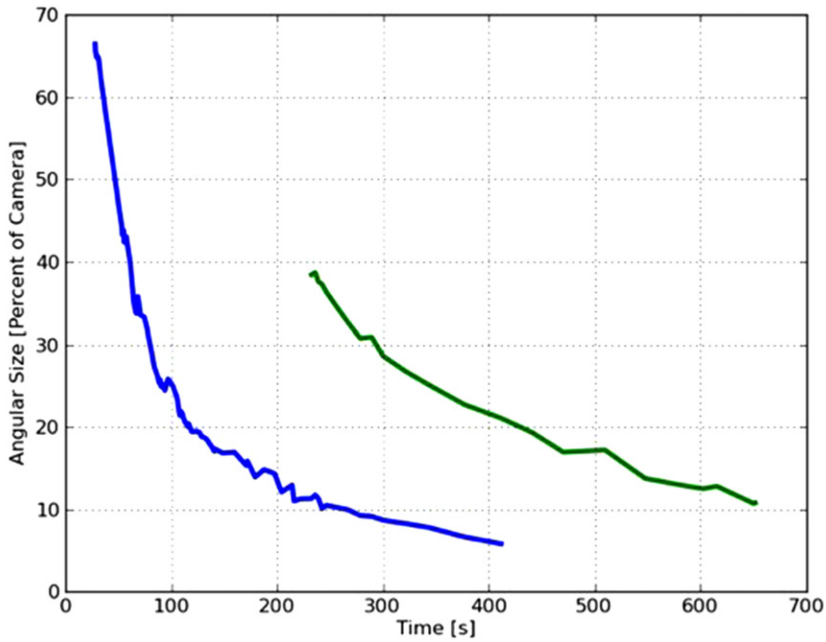

Figure 5.6. Plot of angular size versus time for two objects as seen from the camera.

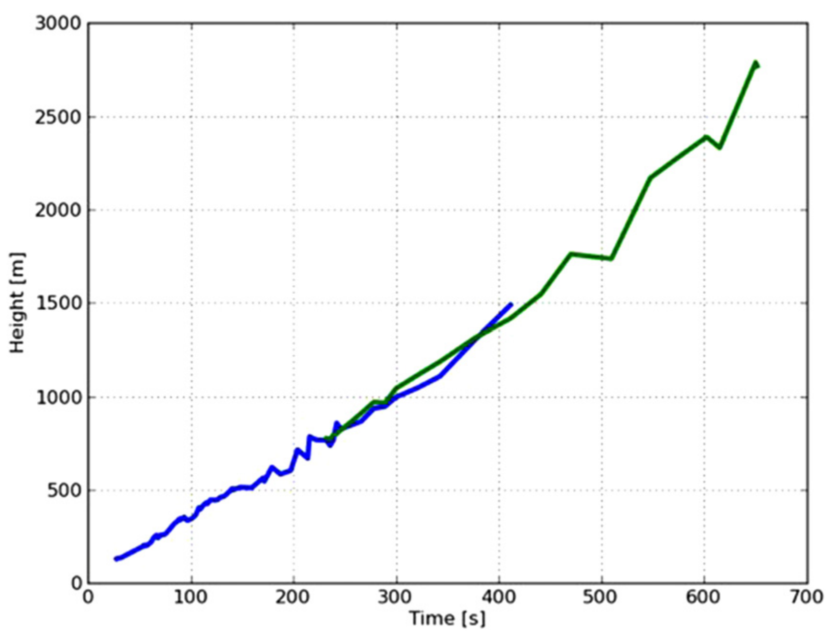

Figure 5.7. Balloon height versus time as calculated from the angular size of the two objects.

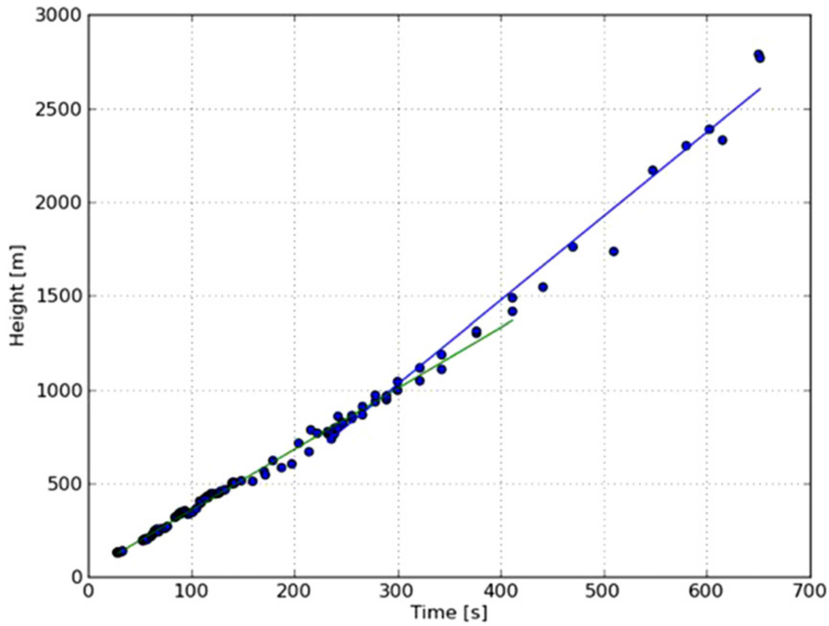

Figure 5.8. Height versus time along with linear fitting equations to estimate balloon velocity.

obtain reasonable position data for an analysis. Since the object's angular size changes with distance we have used the angular size to estimate distance. This is a very useful technique since there are many videos that show something just like this.

References

[1] https://quantumprogress.wordpress.com/2012/04/13/who-says-people-have-lost-interest-in-the-space-program

[2] https://prezi.com/niskdh6p1wcu/spacecats-grant-presentation/

Chapter 6

Using an object's shadow to determine position

In chapter 5, I showed a method for determining the position of an object using the angular size of the object. Here is another example of an indirect determination of an object's position. Let us start with this video of Gary Connery in which he jumps out of a helicopter with a wingsuit but without a parachute [1]. In order to prevent a catastrophic collision with the ground, he lands on a giant runway made of cardboard boxes.

Figure 6.1 shows part of the landing.

This view of the landing is from a camera mounted to another skydiver (one with a parachute). Although the camera view moves around a little bit, it still offers an excellent opportunity to analyze the motion during the landing. By looking at the position of the jumper with respect to the boxes, we can obtain an estimate of his

Figure 6.1. A view of Gary Connery as he lands on a runway made of cardboard boxes. Animated gif based upon [1]. Gary Connery Archive/Getty Images with permission.

doi:10.1088/978-1-6817-4067-6ch6

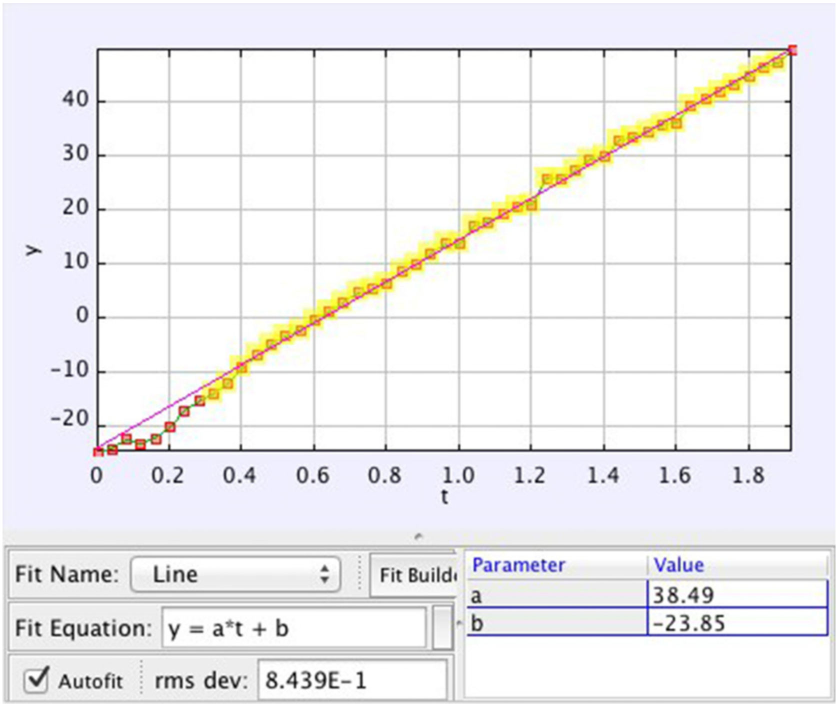

Figure 6.2. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the position of the jumper in the direction of the runway along with a linear fit to estimate the velocity.

forward speed. The length of the runway was reported to be 300 feet. Using that to scale the video, I can obtain the data shown in figure 6.2 for his horizontal position.

Fitting a linear function to these data gives a horizontal landing speed of 38.5 m s^{-1} (86 mph). That was a simple analysis, though. If we want to really examine the landing on the boxes, we also need his vertical motion. This is a bit trickier (and more fun). In order to obtain information regarding his vertical position, we will use the location of his shadow.

Suppose we looked at the jumper from a head-on view at some point during the fall. I could draw the skydiver along with his shadow on the ground. It might look something like figure 6.3.

If I can find the distance x and I know the angle θ, I can use this to find the height y:

$$\tan\theta = \frac{x}{y}$$

$$y = \frac{x}{\tan\theta}. \tag{6.1}$$

If the angle of the shadow is constant and the ground is flat then the height of the jumper is proportional to the distance from the shadow to a point directly underneath the jumper. First, I need the angle of the shadow. By assuming they are 3.66 m high and measuring a box shadow length of 4.06 m, I obtain a shadow angle of 48°.

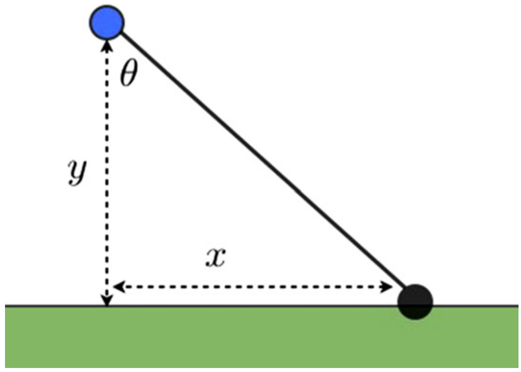

Figure 6.3. Illustration of a jumper and his shadow as it can be used to estimate vertical height.

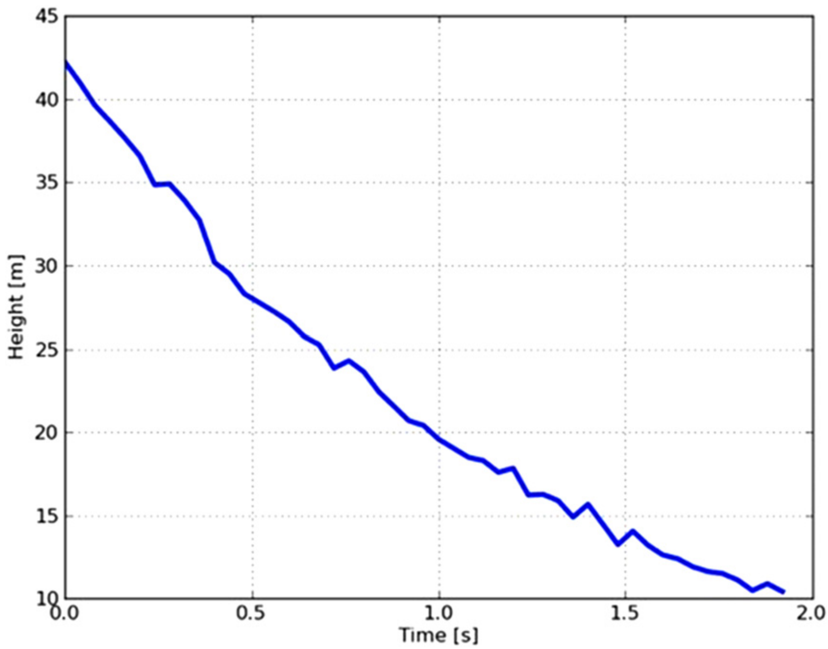

Figure 6.4. A plot of vertical height versus time based on shadow location.

Now I can measure the shadow of Gary Connery to obtain a plot of his vertical position (figure 6.4).

These are only data for the time the shadow was on the ground and not the time the shadow moved on top of the boxes. Here is the last part of the landing with the shadow on the boxes. The plot in figure 6.5 shows the position of the shadow, not the vertical position of Gary Connery.

Converting this to his vertical velocity, I obtain a vertical speed before impact of around 8.68 m s^{-1}.

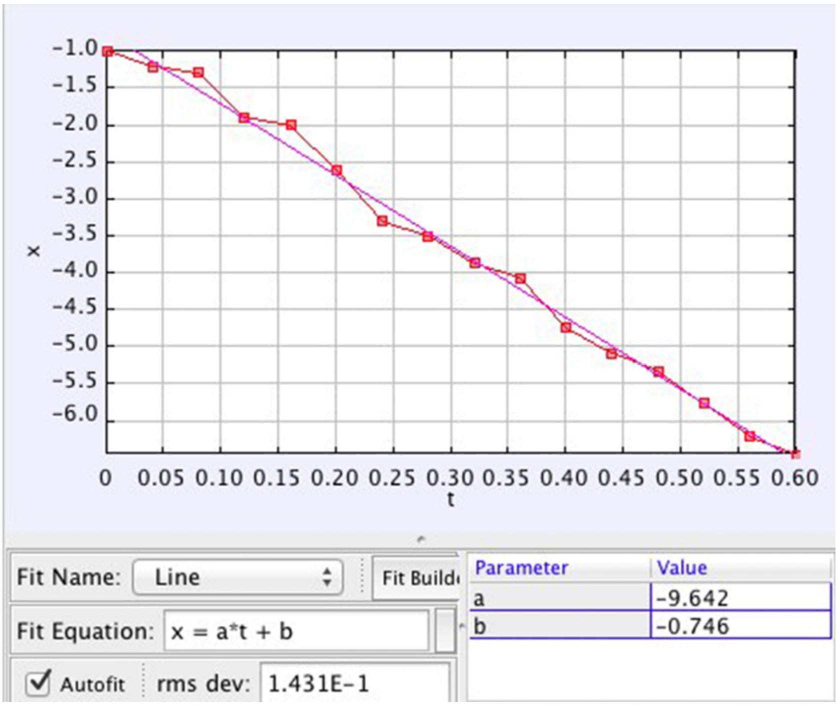

Figure 6.5. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the position of the shadow as a function of time.

As you can see, the vertical speed is significantly lower than you would expect for a human jumping out of an airplane (or a helicopter in this case). The reason for the lower speed is that Gary Connery was using a wingsuit. In some ways this makes him almost like an airplane and allows him to adjust his speed before landing—just like an airplane would. Of course the real lesson is that you do not always need to have direct video evidence in order to analyze a video.

This shows another example of extracting position data from a video that seems like it is not appropriate for a video analysis. Although most of the video would not work, you just need to look for some aspect that will allow you to get the data that you want. Since the shadow moves along the ground, it can give position data. Also, assuming the skydiver moves in a straight line, the distance from the shadow to a point beneath the jumper depends on his height.

Reference

[1] www.youtube.com/watch?v=riK2350eemM#t=130

Chapter 7

Detecting fake videos

Let us be honest. The internet is full of videos and many of these videos are not real. Sometimes they are not supposed to be interpreted as a real video (like *Star Wars* videos or game videos). Other times, people create fake videos to trick other humans into believing they are real. Why do they do this? I do not know. Finally, there are some videos that seem so incredible that we think they are fake, but perhaps they are not. Thus we are left with a question—is that video real or fake?

Method 1. Unrealistic trajectories

There are countless videos online that show someone or something being launched through the air. Maybe it is a person jumping off a building or a car driving over a ramp. But in all of these cases, there is most likely just one force acting on the object—gravitational force.

Objects with only a gravitational force (and near the surface of the Earth) will have a constant vertical acceleration. We call these motions 'projectile motion' and they would have the following two equations for the horizontal and vertical directions:

$$\begin{aligned} y &= y_0 + v_{y0}t - \frac{1}{2}gt^2 \\ x &= x_0 + v_{x0}t. \end{aligned} \tag{7.1}$$

So, if I have a video showing projectile motion and it does not have a constant vertical acceleration and a constant horizontal velocity, then it is probably fake. Of course, there could be cases with significant air resistance, but for most cases this is an insignificant force.

How about an example? Look at figure 7.1, based on a video from the internet. Yes: it shows a buggy driving on the Moon. The Moon! Could that be real? It seems unlikely, does it not? Let us find out. As I said before, we need an object that only has gravitational force on it. In this case, dust from the tires fits the description. Of

doi:10.1088/978-1-6817-4067-6ch7

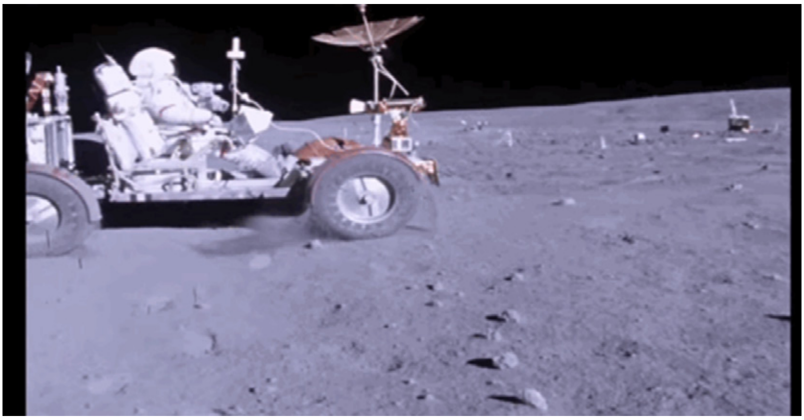

Figure 7.1. Animated gif showing the motion of a lunar buggy, based upon [1].

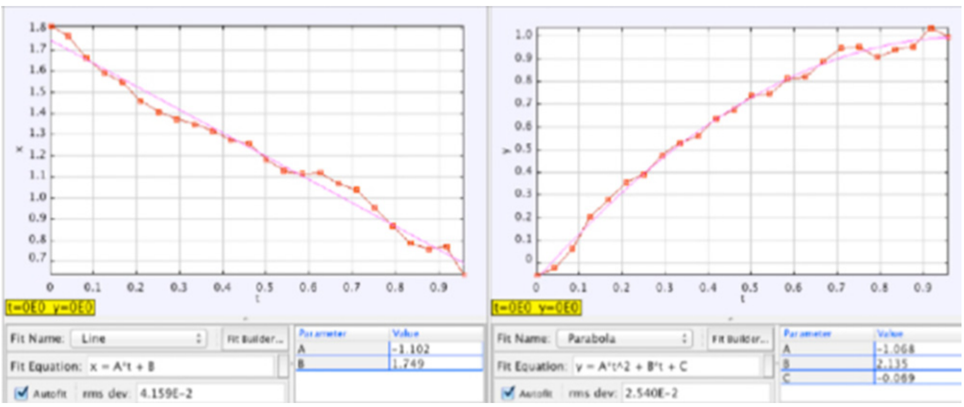

Figure 7.2. Screen capture from Tracker Video Analysis showing the horizontal and vertical motion of moon dust.

course, dust on Earth does not fall in line with projectile motion, but on the Moon there is no air to keep the dust up.

Using video analysis, I can obtain the plots in figure 7.2 for the vertical and horizontal motion of some of the dust. The left graph shows that the horizontal velocity of the dust is fairly constant. The right graph shows that the vertical motion of the dust has a constant acceleration of 2.14 m s^{-2}. So, I think this video is real. Humans actually drove a buggy on the Moon. Yes, the accepted value of the acceleration on the surface of the Moon is 1.6 m s^{-2}, but this value is close and constant. There are some issues with the scale of the video, but it is still real.

Figure 7.3 shows another video with projectile motion [2]. Yes, this is an animation so it should be fake. However, that should not stop us from performing an analysis. Especially as it is always said that elephants cannot jump—not even a little bit. For a good part of the video, we have a nice view of the jumping elephant. Looking at the vertical motion, we obtain a plot like that in figure 7.4.

Figure 7.3. Animated gif showing an elephant jumping on a trampoline, based upon [2]. Reproduced by permission of Nicholas Deveaux, Cube Creative Computer Company.

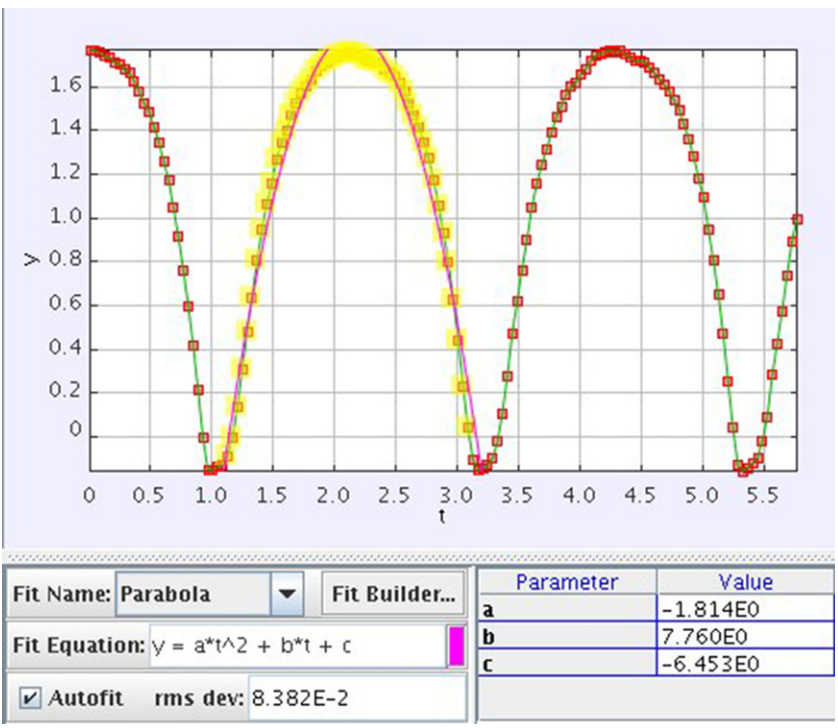

Figure 7.4. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the vertical position of the jumping elephant.

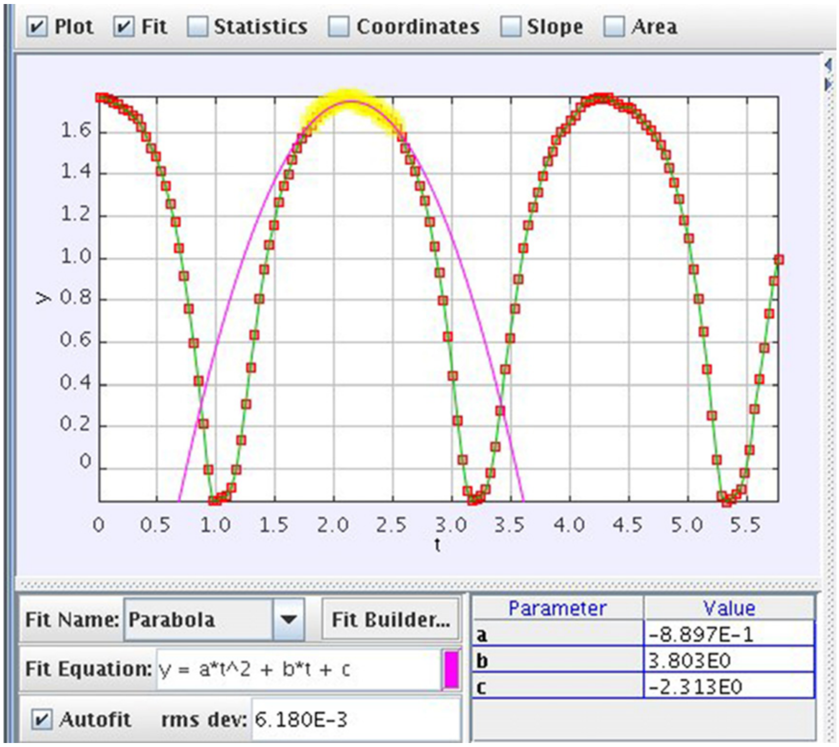

Figure 7.5. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the vertical position of the jumping elephant along with a quadratic fit to part of the data.

What about the scale? I used the diameter of the trampoline as one unit of distance. That means the vertical axis is not in meters but in 'trampolines'. Now, if I fit a quadratic function to the second bounce I can find the acceleration. This gives a value of -3.6 trampolines s^{-2}. However, when you look at the fitting function, it does match up very well with the data at the top of the graph.

If I just fit a quadratic function to the highest part of the jump, I obtain figure 7.5. This gives an acceleration of -1.8 trampolines s^{-2}. I suppose that the creators of the movie wanted to add emphasis to the motion at the highest point by making it take longer. But this shows an important point about animations. Often it is easier to use real physics to model the motion of something like a bouncing elephant. However, it is still an artistic creation in which the author can adjust some of the parameters to achieve the final result in a certain way. In this case, the animator chose two different accelerations for the bouncing elephant.

Method 2. Impossible physics

What if I do not want a frame-by-frame analysis of motion in a video? Instead, suppose that I just look at the video to see if it is even possible based on known laws of physics.

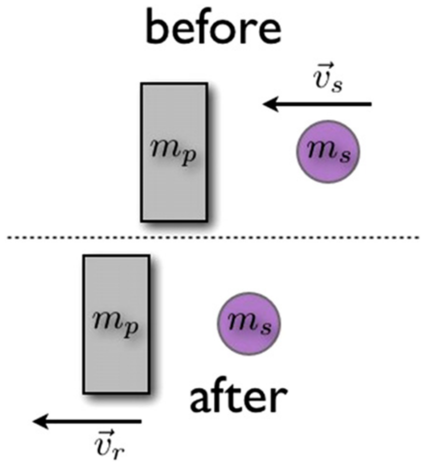

Figure 7.6. Diagram showing the foam and the person both before and after the collision.

Look at this video [3] showing a man in a lab being hit by styrofoam shot from a pneumatic cannon. From the video, I can obtain a recoil speed for the lab of about 10 m s^{-1} (ignoring for a moment what I said about not looking at it frame-by-frame to obtain data). If I estimate the mass of the person and the foam projectiles, I can treat this as a standard collision (and conservation of momentum) problem, as seen in figure 7.6.

If the foam has a mass of 5 kg, it would have to come out of the launcher with a speed of 130 m s^{-1} (290 mph) in order to achieve the kind of recoil shown in the video. What is even worse is that the man who was hit would have an acceleration of at least 14.6 g. So, if this was indeed a genuine prank it was one that was not very funny. But it is fake.

Figure 7.7 shows another example. In this video, there is a trick with a glass of water. You fill the glass up all the way (with distilled water for the best effect). Next you use a temporary sheet of paper so that you can get the glass upside down on a flat surface while still filled with water. Now just spin the glass and lift. This is what happens—at least in the video.

Why is this fake? It is fake because it shows impossible physics. Let us start with a force diagram showing water in an upside down glass (but not spinning). Figure 7.8 just shows the forces on a small part of the water. First, let me just look at the vertical forces. Of course there is the gravitational force on this piece of water pulling down. Why does this block of water not move? Because of the net force from the other water pushing on it. The water below it pushes up more than the water above it pushes down. Why? Because the pressure increases as you go deeper in the water. Since I have a cubical block of water with the same area on the top and bottom, greater pressure means greater force. This is why things float in water (and it does not have to have the same sized areas on the top and bottom).

What about the horizontal forces? Well, since the right side of the water block is underwater, there is a force pushing to the left from the water. The water block is in equilibrium because the side of the glass also pushes to the right with the same force

Figure 7.7. Animated gif showing a trick with a spinning glass of water, based upon [4]. Reproduced with permission from Dan DeEntrement.

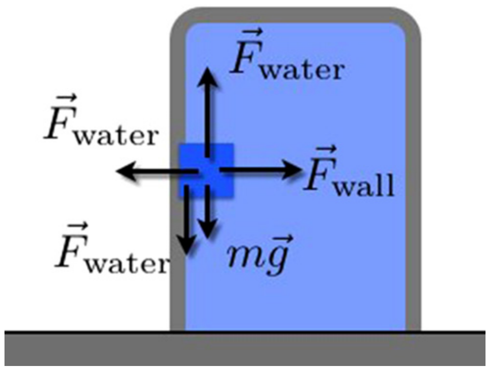

Figure 7.8. Diagram showing the forces on a section of water in a spinning glass.

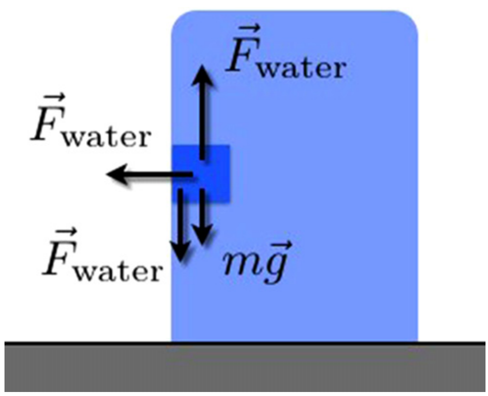

Figure 7.9. Diagram showing the forces on a section of spinning water without the glass.

as the water pressure force. It seems simple, does it not? (Note that I picked a section of water on the side so that we could see what happens when the glass is removed.)

Now take away the glass with non-spinning water. What changes? Figure 7.9 shows another diagram. The only changes are the absence of the glass and the force from the wall. But now there is only one force pushing horizontally. How can this water be in equilibrium? It cannot. But what about spinning the water? Now that same piece of water will be moving in a circle and thus accelerating towards the center of the circle. Since the acceleration of this piece of water is towards the center of the circle, there must also be a force in this direction. Now that the force from the wall of the glass is gone, what force makes it move in a circle? There is no such force. Spinning would make the water fly apart faster, not stay together. The video is, of course, fake.

Method 3. Detecting fake shake

Here is the basic idea behind 'fake shake'.

- Someone wants to make a 'cool' fake video.
- How do you record a fake video? One method is to use a camera on a tripod. This makes the background stationary so that you can later add fake video effects.
- Of course, a real video of something 'magically' fake would not be recorded with a tripod. This can be fixed by making the video appear to shake as though it were being held by a human.

That is it. A fake video with fake camera shake. Can we detect the fake shake? Figure 7.10 shows a short example of a suitable video. There are two things to note in this video. First, note that this seems fantastically impossible. How could you hit a

Figure 7.10. Animated gif showing a baseball rebounder trick, based upon [5]. Reproduced with permission from Easton.

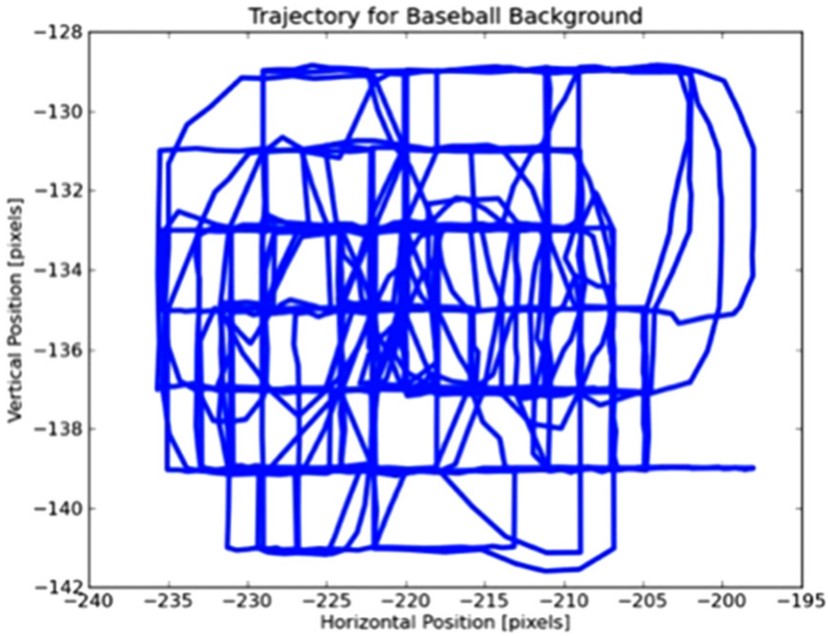

Figure 7.11. Trajectory of the background motion of the baseball trick video.

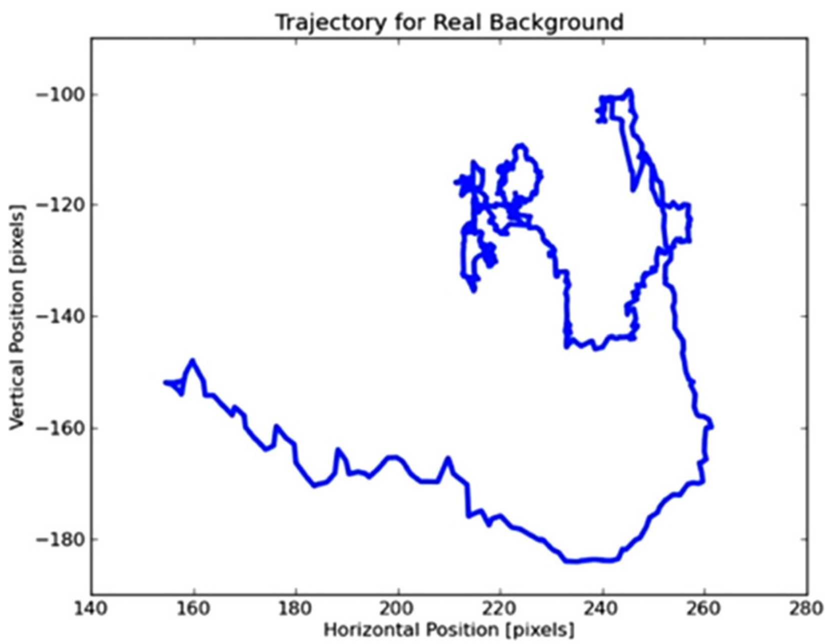

Figure 7.12. Trajectory of the background motion for a handheld camera.

baseball with a bat so precisely that it would rebound off four springs and come back to the batter? Second, there is an ever-so-slight shake to the camera.

Normally in video analysis, we are interested in the motion of an object. Here, we are interested in the motion of the background. If I pick some feature of the background to track, I obtain the trajectory plot of the background shown in figure 7.11.

Is this evidence that the video is fake? Let us make a quick comparison. Suppose I do something similar with a real video camera. Figure 7.12 shows the trajectory background plot for a camera that I held as stationary as possible. Notice the difference? The fake video has regular looking 'shakes', whereas the real video looks more random—actually much like a random walk. Detecting fake shake does not prove a video is fake, but it seems to be very convincing evidence.

Those are the three most useful methods for detecting fake videos. Personally, if the video appears to possibly be fake and recorded without a tripod I go straight for the fake shake detection method. It is the easiest to check. After that, you can look for possible problems with the physics in the video.

References

[1] www.youtube.com/watch?v=7o3Oi9JWsyM

[2] http://youtu.be/xdufuHcg4fY

[3] www.youtube.com/watch?v=wmM00LDnLBI

[4] www.youtube.com/watch?v=7ctaA2mERzI

[5] www.youtube.com/watch?v=W8SK0rk5jdE

Chapter 8

Video analysis of spin during the Red Bull Stratos jump

In itself, the Red Bull Stratos jump was impressive. The stunt was performed by skydiver Felix Baumgartner riding in a balloon up to an altitude of 128 000 feet. From that height, he then jumped out (wearing a pressure suit, of course). During the fall, he travelled faster than the speed of sound because of the low density of air. Finally, he landed safely. It was an historic event.

Although the jump ended well, there was one tense part of free fall. While still high in the atmosphere, Felix began to spin. The spin was induced by a torque exerted from the air due to a non-symmetrical body position. Since the density of air was low, it was difficult for Felix to use normal skydiving moves to counteract spin. In the end, Felix did finally manage to reduce the spinning rate as he entered the denser atmosphere.

Here you can view a short clip showing the spin [1]. Of course the real question is: how bad was this spin? How many g forces did Felix experience during this part? Is it possible to even estimate this value?

Using video analysis, we can obtain an estimate for his rotational rate during the fall. In this case, I will mark the position of the jumper's head and a separate position for his feet. This way I can look at the rotational motion independently of the motion of camera. Really, this is the only option, as there is nothing in the background that would allow me to account for the camera motion.

Figure 8.1 shows a plot of the horizontal value of the head position relative to the feet position. The data for this plot are in pixels. Since I am only interested in the rotation rate, the distance scale does not really matter. Looking at the plot above, Felix made four complete rotations in 3.44 s. This gives a rotational speed (angular velocity) of:

$$\omega = \frac{8\pi}{3.44\ \text{s}} = 7.31\ \text{rad s}^{-1}. \qquad (8.1)$$

What kind of G force would this produce? First, let me assume that he spins about his center of mass. Figure 8.2 contains a diagram of this. I have a value for the

doi:10.1088/978-1-6817-4067-6ch8

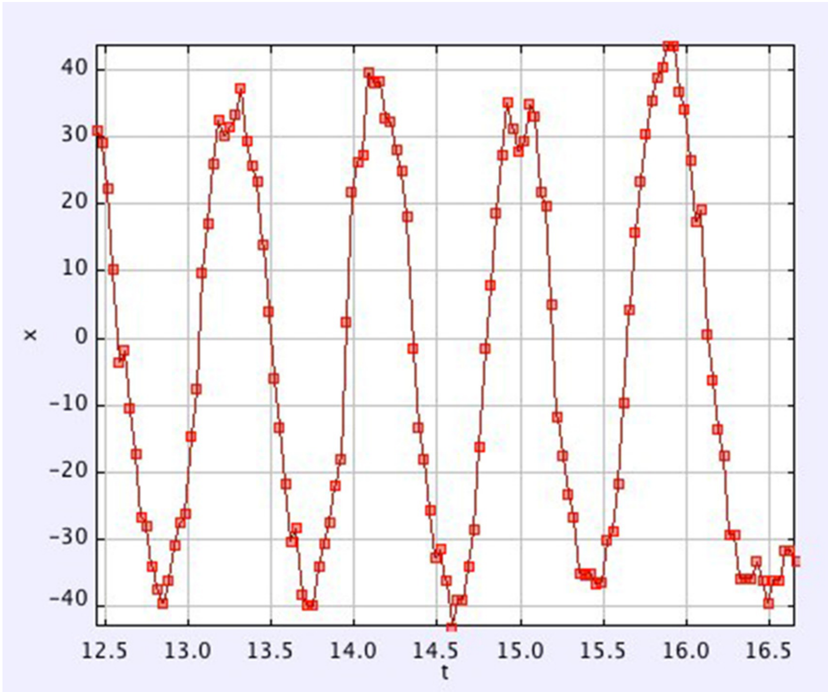

Figure 8.1. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the horizontal position of the jumper's body as he spins.

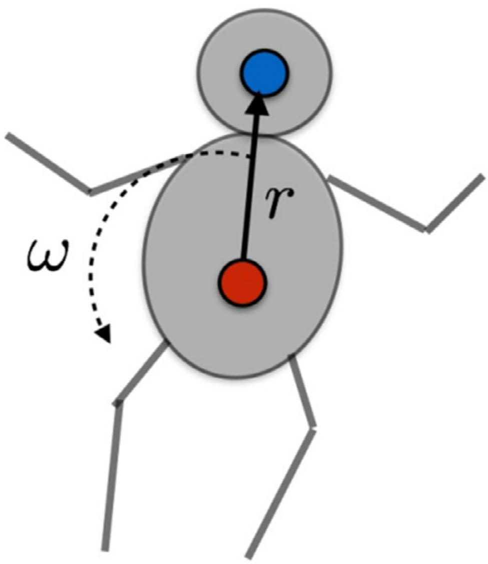

Figure 8.2. Diagram showing the location of two points used to measure the spin of the jumper.

angular speed (ω) but what about the value of r? First, this assumes that the center of rotation is at the jumper's center of mass. Second, I can estimate the distance from the center to the head as a distance of about 0.75 m. With that, I can obtain a value for the acceleration of his head. Recall that if an object (his head) is moving in a circle it accelerates even if moving at a constant speed because of the constant change in direction.

In terms of the angular velocity, the acceleration would be:

$$a = \omega^2 r. \tag{8.2}$$

With an angular acceleration of 7.31 rad s^{-2} and a radius of 0.75 m this gives an acceleration of 40.1 m s^{-2} or about 4 g. From what I can guess, that would not be much fun. Also, since his head is accelerating towards his center of mass there would be a fake centrifugal force pushing blood towards his head. According to Wikipedia's entry on G-force tolerance, a person can only take about -3 g before a red out [2]. This is where excess blood pressure in the capillaries causes vision problems.

This is another example of a video where the data are not very straightforward. As always, you want to look for some part of the video that could have a meaningful significance. In this case, it was the angular velocity of the jumper.

References

[1] www.youtube.com/watch?v=dOoHArAzdug
[2] http://en.wikipedia.org/wiki/G-force

Chapter 9

Analysis of an NFL flop

Flopping occurs in just about every sport (e.g. basketball, soccer and American football). A flop is when a player adjusts his own motion to create the appearance of a push from an opposing player. This video shows a great example from the NFL in which Jerome Simpson falls down [1].

Here you can see that Scott Fujita (of the Cleveland Browns) appears to push Simpson (of the Cincinnati Bengals) in such a manner as to send him flying backwards. The actions of Fujita prompted a penalty from the referees.

Before getting into a video analysis of this motion, let us take a look at the interaction between these two players. If Fujita pushes on Simpson, then the horizontal momentum should be mostly conserved. Why should momentum be conserved? Figure 9.1 contains a diagram showing the two players during the interaction.

Looking from a physics view, it does not matter which player pushed. What matters is that this force is between the two players and has the same magnitude. The

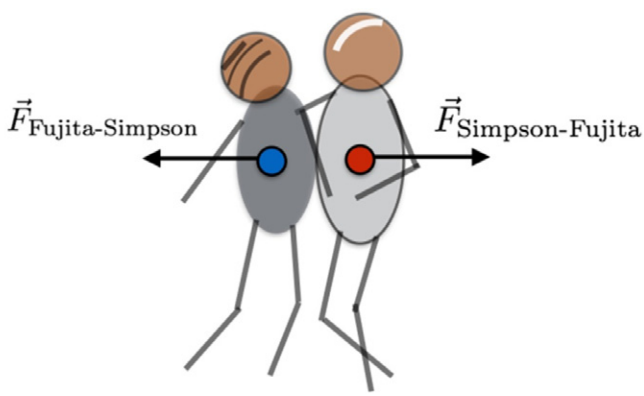

Figure 9.1. Diagram showing the estimated center of mass for both players along with the forces on the two players, based upon [1].

doi:10.1088/978-1-6817-4067-6ch9

force of Fujita pushing on Simpson is the same as Simpson pushing on Fujita (but in the opposite direction). If I assume no other external forces (no gravity and no force from the ground), then the momentum principle says:

$$\begin{aligned}\Delta\vec{p}_{\text{Fujita}} &= \vec{F}_{\text{Simpson-Fujita}}\,\Delta t\\ \Delta\vec{p}_{\text{Simpson}} &= \vec{F}_{\text{Fujita-Simpson}}\,\Delta t\\ \Delta\vec{p}_{\text{Fujita}} &= -\Delta\vec{p}_{\text{Simpson}}.\end{aligned} \tag{9.1}$$

This is basically conservation of momentum. The momentum before and after the 'altercation' would be the same if you ignore the players pushing on the ground. For gravity, you can just look at the changes in momentum in the horizontal direction. Gravity does not pull this way, so it would not have any effect on the horizontal momentum.

If you look at the video closely, you will see that Fujita is in the air for a good part of the interaction. If that is the case, the horizontal momentum should be conserved, assuming that Simpson is not pushing on the ground either.

Now that we know that horizontal momentum should be conserved, we can look at the motion of both Fujita and Simpson during the interaction. Using video analysis, I can get the plot shown in figure 9.2 for Fujita's position both before and after the 'push'.

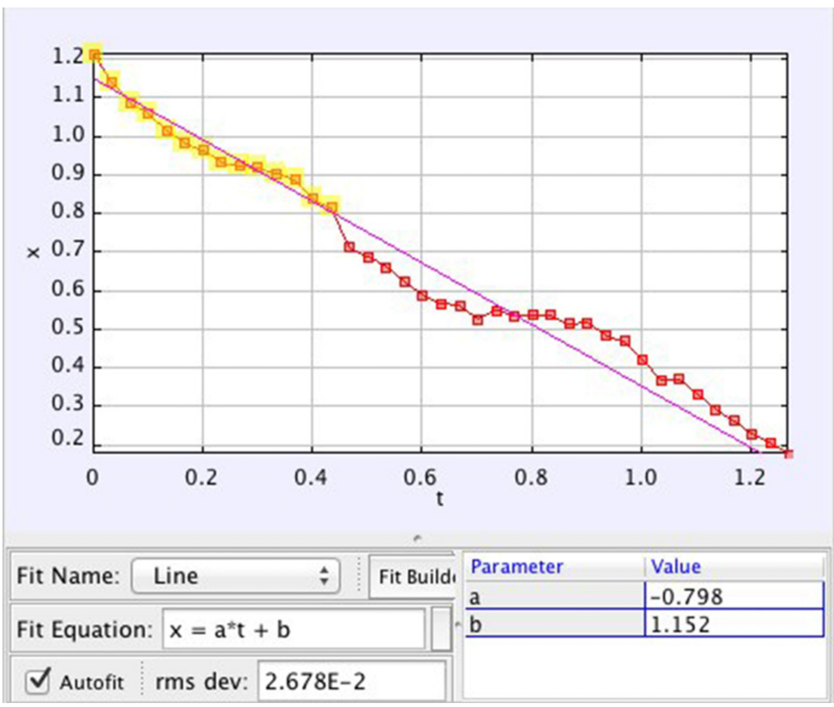

Figure 9.2. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the position of Fujita.

Yes, Fujita does not have a constant velocity before the altercation. He is getting up and pushing with his feet, which means that he would accelerate a little bit. However, his velocity seems constant enough for me to say he was moving at about 0.8 m s^{-1}. What about after the interaction? Afterward he is moving with around the same horizontal speed—about 0.9 m s^{-1}. Now, what about Simpson? The plot for his position in the direction of motion is given in figure 9.3.

Simpson was moving toward Fujita and then afterward was moving away. His first horizontal speed was around 0.9 m s^{-1} (to the right) and his speed afterward was around 2.2 m s^{-1} (to the left).

Now for the momentum. Wikipedia lists Fujita's mass at 113 kg [2] and Simpson's at 88 kg. This would put the total before-and-after horizontal momentum (of the system consisting of both players) at:

$$
\begin{aligned}
p_{x1} &= m_F v_{Fx1} + m_S v_{Sx1} \\
p_{x1} &= (113\ \text{kg})\left(-0.8\ \text{ms}^{-1}\right) + (88\ \text{kg})\left(0.9\ \text{ms}^{-1}\right) = -11.2\ \text{kg*ms}^{-1} \\
p_{x2} &= (113\ \text{kg})\left(-0.9\ \text{ms}^{-1}\right) + (88\ \text{kg})\left(-2.2\ \text{ms}^{-1}\right) = 295.3\ \text{kg*ms}^{-1}.
\end{aligned}
\tag{9.2}
$$

What does this say? This says that momentum is not conserved if you only consider the interaction between the players. What could have happened, then? There are two possible options.

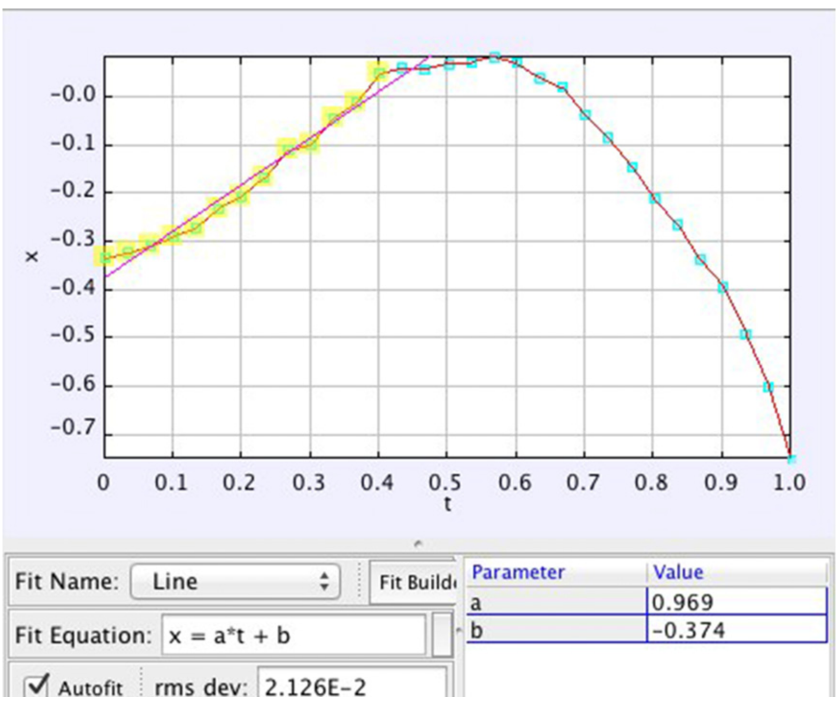

Figure 9.3. Screen capture from Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the position of Simpson.

- Fujita was pushing on the ground (giving an external force) during the interaction. This does not seem very likely since he was not touching the ground the whole time.
- Simpson is faking. He pushes off the ground to make it look like Fujita body-slammed him.

It *is* possible that Fujita pushed Simpson. Well, he clearly pushed—but did he push that hard? We can obtain another estimate for the strength of the push. What is the change in momentum for Simpson?

$$\Delta p_{xS} = (88\ \text{kg})\left(-2.2\ \text{ms}^{-1} - 0.9\ \text{ms}^{-1}\right) = 272.8\ \text{kg*ms}^{-1}. \tag{9.3}$$

Since Fujita's change in momentum is essentially zero, this change in Simpson's momentum must be from the force exerted by Fujita on the ground (and the ground on Fujita). From the video, I estimate that he was in contact with the ground *and* Simpson for about 0.3 s. This net horizontal force on Simpson would then be:

$$F_{\text{net-}x\text{-Simpson}} = \frac{\Delta p_{xS}}{\Delta t} = \frac{272.8\ \text{kg*ms}^{-1}}{0.3\ \text{s}} = 909\ \text{N}. \tag{9.4}$$

What would be the effective coefficient of friction for Fujita's shoes to pull this off—or maybe I should say, push this off? If I do not consider that Fujita is pushing up on

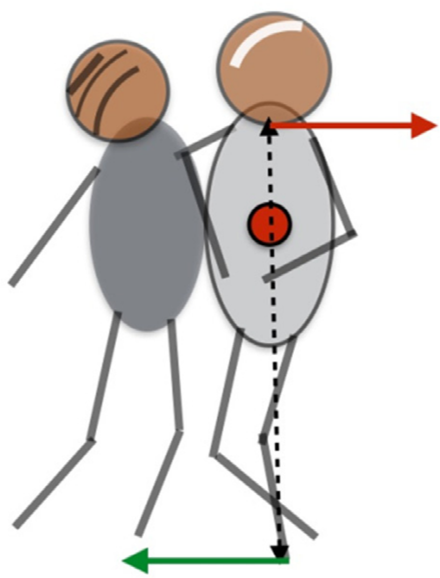

Figure 9.4. Diagram showing the forces and torque on Fujita during the push.

Simpson, then the vertical component of the ground on Fujita would just be his weight (113 kg*9.8 N kg^{-1} = 1107 N). This would give an effective coefficient of:

$$F_{\text{friction-max}} = \mu_s N$$
$$\mu_s = \frac{909\text{N}}{1107\text{N}} = 0.82. \tag{9.5}$$

A coefficient of 0.82 may seem large, but remember that NFL players wear cleats, so it is indeed possible. So was it possible that the fall backwards of Simpson was caused by Fujita? There is something else to consider. If Fujita did indeed push on Simpson with a force of 909 Newtons, then Simpson would have to push back with a force of 909 Newtons on Fujita due to the nature of forces. This would mean that Fujita could have a net force of zero and no change in momentum—which is how it appears.

But as we have already seen, Fujita does not appear to be pushing on the ground such that there is no frictional force. This means that Fujita's change in momentum should be the opposite of Simpson's change in momentum. With their different masses, Fujita would have a recoil speed of 1.6 m s^{-1} if he were the cause of Simpson's fall (which he clearly did not).

There is one other consideration: torque and angular momentum. If Fujita pushed on Simpson while also pushing on the ground, both of these forces would exert a torque about his center of mass (figure 9.4).

Although these forces would add up to zero net force, the sum of the torques is not zero. These forces would both produce a clockwise change in angular momentum. If Fujita were responsible for Simpson's motion, you would see him 'falling back' after the shove. In fact, Fujita seems to remain fairly upright during the whole process and does not tip back.

In the end, it seems the evidence is clear. Simpson probably flopped.

References

[1] www.youtube.com/watch?v=pEcQhrsc8JE

[2] http://en.wikipedia.org/wiki/Scott_Fujita

[3] http://en.wikipedia.org/wiki/Jerome_Simpson

Rhett Allain

Chapter 10

Running the loop the loop

Maybe you have seen a car driving in a circular loop. It is a good stunt. But what about a human running around a vertical loop? In [1], Damien Walters does exactly that in this video.

When people see a stunt like this, they often think 'how is this possible?' It does indeed seem almost magical. The human running the loop appears to be pressed against the top of the track. It is clear that gravity pulls him down, but what pushes him up? One answer is that the centrifugal force pushes him away from the center of the circle.

Perhaps you learned that centrifugal force was a 'bad thing' in introductory physics courses. That is only partially true. The centrifugal forces are a type of fake force. The problem comes about when students confuse real and fake forces. But if you know it is a fake force, it can be useful.

But what is a fake force? Let us step back and think about the nature of force and motion. If I have a net force on an object, I can write the following:

$$\vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t} = m\vec{a}. \tag{10.1}$$

However, this only works when the acceleration (or change in momentum) is measured from a reference frame that is not accelerating. If you want to use an accelerating reference frame instead (like following the human as he moves in a circle), then you need to add a force to make the momentum principle work again. Using this fake force, the human would have three forces acting on him (figure 10.1).

At the top (and in the frame of the runner), these three forces have to add up to zero (zero vector). At the very lowest speed, the force of the track would also be zero so that the fake force and gravitational force balance. But how do we find the fake force? It is just the negative of the product of the mass of the object and the acceleration of the frame:

doi:10.1088/978-1-6817-4067-6ch10

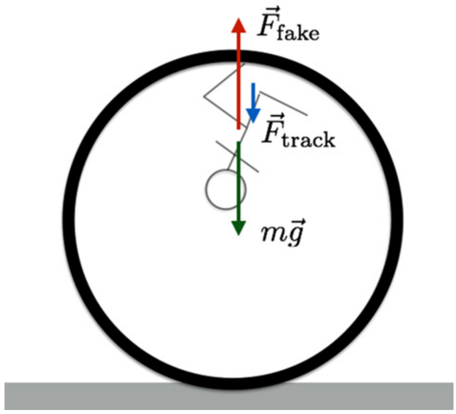

Figure 10.1. Diagram showing the forces on a human running around a vertical loop.

$$\vec{F}_{\text{fake}} = -m\vec{a}_{\text{frame}}. \tag{10.2}$$

Using the acceleration of an object moving in a circle, I can write this force equation (in just the vertical direction):

$$F_{\text{fake}} - mg = 0$$
$$a_c = \frac{v^2}{r}$$
$$\frac{mv^2}{r} = mg. \tag{10.3}$$

However, there is something significantly different about a running human and a car, especially with this size of a loop. Different parts of the human are moving in different sized circles. In fact, if the human's head extends past the center of the circle, there will be a fake force pulling *down* on that part of the body.

So, what about the calculation in the video? The female in that clip suggests that you only need to be moving at 8.65 mph to complete the loop move (3.87 m s^{-1}). Let me calculate the speed that a small car would need to go around this loop (since the size of the car is small compared to the radius of the circle). I can use this image to estimate the size of the loop.

The runner is Damien Walters with a listed height of 1.8 m. From this I obtain a circular radius of about 1.4 m. Using the above equation, I just need to solve for the velocity when there is no force from the track:

$$\frac{mv^2}{r} = mv$$
$$v = \sqrt{gr} = \sqrt{(9.8\ \ \text{N/kg})(1.4\,\text{m})} = 3.7\,\text{ms}^{-1}. \tag{10.4}$$

So, that is basically the value they calculated in the video. I am fairly sure this is too low. The fake force pushing the runner up will be much smaller than it needs to be at that speed. Why? Well, only the person's feet are at that radius. The rest of the person is closer to the center. You would have to run much faster.

Also, this is the speed at the top. As a human runs up the side, the human would also need to work against gravity (or start with an even higher speed). How do I know the calculated value in the video is wrong? Well, I can probably run 3.7 m s^{-1} and I am almost certain I could not do this stunt.

How would you calculate the minimum speed? I think you would need to calculate the differential fake force on different parts of the body. Then you could integrate over the volume of the human body to obtain the total fake force. However, since the calculation of a differential fake force does not involve video analysis, I will just skip to an analysis of the loop run.

This video is almost perfect for analysis. The only negative aspect is that the camera moves a little bit, so it is just a nice video and not perfect. Figure 10.2 shows the trajectory (x versus y) for a point on his waist after correcting for camera motion.

Maybe it is more useful to look at the angular position of the waist as he runs the loop (figure 10.3). The slope of this graph (just what happens after 0.5 s) gives an angular velocity of 4.805 rad s^{-1}. If I know the radius of motion, I can find the speed using the relationship between the angular and linear speeds for an object moving in a circle:

$$v = \omega r. \tag{10.5}$$

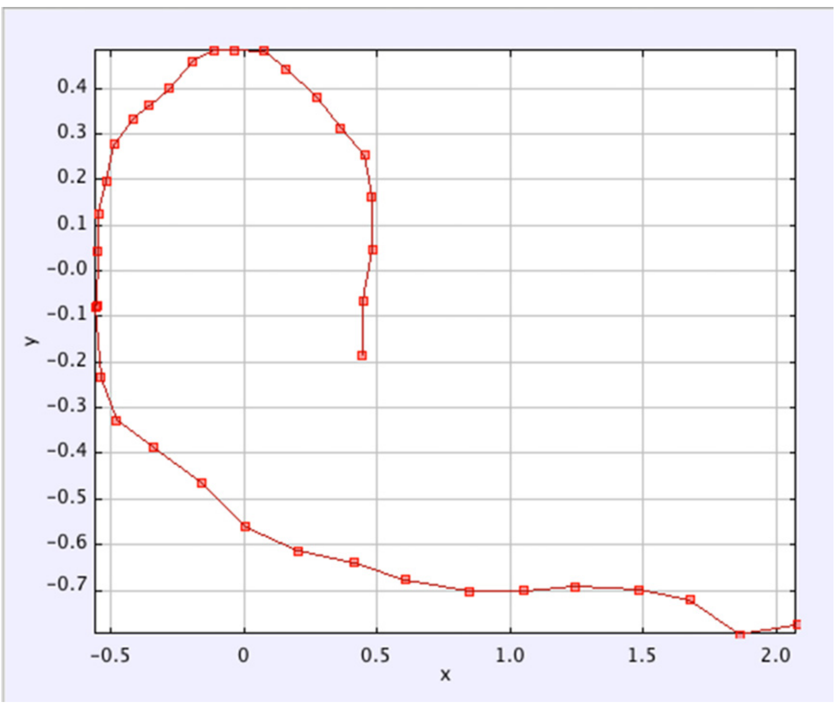

Figure 10.2. Screen capture from the Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the trajectory of runner's waist as he runs around the vertical loop.

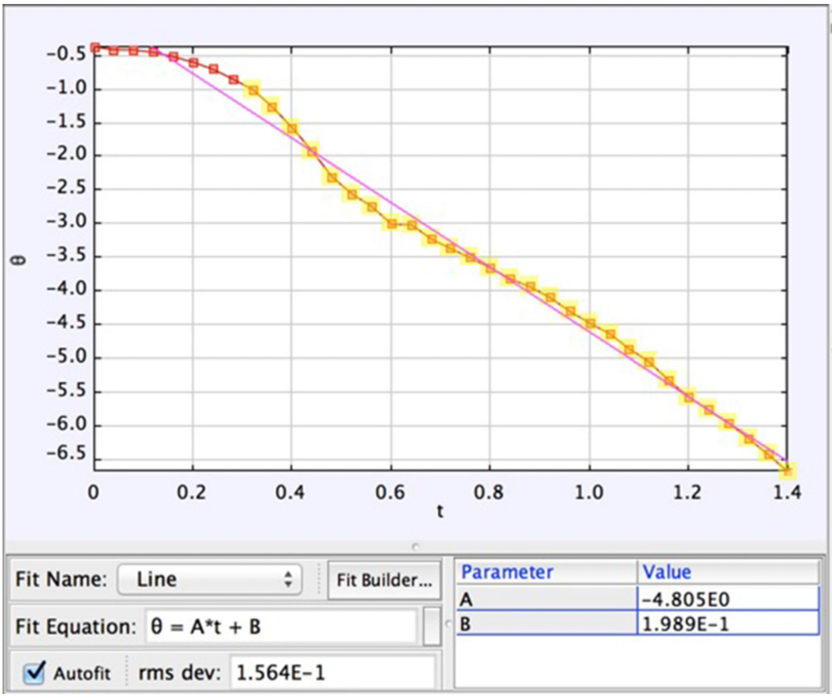

Figure 10.3. Screen capture from the Tracker Video Analysis (Software made available under the GNU General Public Licence, version 3 (GPL-3.0).) showing the angular position of runner as he runs around the vertical loop.

Since his waist is about 0.5 m from the center of the loop, the velocity of his center of mass would be 2.40 m s^{-1}. The speed at the outer part of the loop would be 6.727 m s^{-1} (15.0 mph). So, what does this all mean? Well, the first thing to note is that his linear speed is different for different parts of his body. In fact, if you look at the video carefully, his head is actually going in the opposite direction to the rest of his body at one point. This is because he is doing two things. He is moving in a circle *and* his body is rotating.

If you go back to the original calculation of necessary speed, the runner's feet are going at a much greater speed than the suggested 8.65 mph.

This video is a great example of a case where it is all too difficult to obtain position data. However, once you get those data the analysis can be quite interesting.

Reference

[1] www.youtube.com/watch?v=OTcdutIcEJ4

Chapter 11

Video analysis of the speeder in the *Star Wars VII* trailer

One of my favorite activities is to take the trailer for a yet to be released movie and analyze it. In this case, I am looking at the trailer for *Star Wars VII: The Force Awakens*, which at the time of writing has not yet been released. I have no idea what is actually going on in the trailer, but there is one scene that shows a female on some type of speeder. Well, I do know something, I know that the character riding the speeder is named Rey and is played by Daisy Ridley. Other than that, I am going to have to figure out some things in order to obtain a measurement of the speed (and acceleration) of this vehicle.

This is not such an easy scene to analyze. The motion of the speeder starts off perpendicular to the camera, but then quickly moves away as the camera pans. Normally I would use a measure of the angular size of the speeder to plot the position. However, since it moves away at an angle this is not very easy.

The first thing I need is an estimate for the length of this speeder. According to Wikipedia [1], Daisy Ridley is 1.7 m tall. With this, I can obtain a size estimate for the speeder. The main long part of this speeder is 2.8 m.

If the object was moving straight away from the camera, I could determine the distance to the object based on the angular size. However, in this case there is an extra problem. Since the angle of the speeder with respect to the camera changes, I will have to take into account this and the angular size. Suppose I have the speeder at some distance away and tilted at an angle α. In that case, I can find the distance (r) as per figure 11.1.

Since I know L and I can estimate θ, I just need a way to find the angle α. This might be a stretch, but I am going to measure the ratio of the length and width of the speeder in order to obtain the angle. Maybe the diagram in figure 11.2 will help.

doi:10.1088/978-1-6817-4067-6ch11

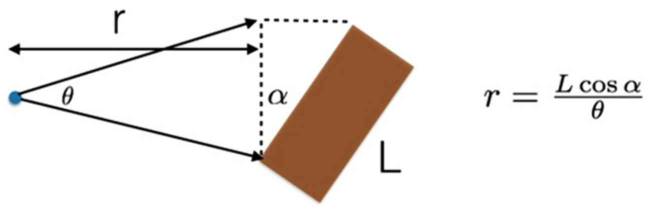

Figure 11.1. Diagram showing the camera angle viewing the speeder.

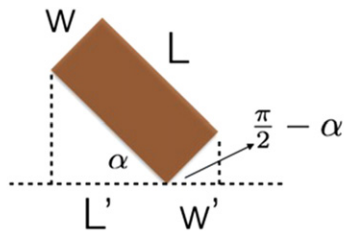

Figure 11.2. Diagram showing the measurable dimensions of the speeder and how they relate to the angle of velocity.

The speeder has a length L and a width W. When it is at an angle, it has an apparent length of L' and apparent width W'. By looking at the ratio of apparent to actual lengths, I obtain the following:

$$\tan\alpha = \frac{w'L}{wL'}. \tag{11.1}$$

I am almost ready to produce some data. I just need one more thing—the angular view of the camera. I can measure the angular size of an object from the video, but I do not know how many pixels in the frame there are for each degree of an angle. Here is where I will make an educated guess. If this was created with a 35 mm camera, then it would probably have an angular field of view of 39.6° in the horizontal dimension. Using this value for the angular field of view, the speeder would start off 8.9 m from the camera. I think that sounds about right.

Now that I have a way to calculate both the angular direction and the angular size of the speeder, I can make some plots. First, figure 11.3 is a plot of the angle the speeder is heading in with respect to the camera as a function of time.

Just to be clear, this is the angle α from figure 11.2. The speeder starts off pointing perpendicularly to the camera and this would be an angle of zero degrees. Also, if you look at the angle plot it is nice and smooth—that is good. As the speeder travels further away, the angle should not change too much, either. Finally, if you look at where this angle is approaching, it seems that I can make the diagram in figure 11.4 that shows the camera and the speeder.

In order to get the x, y position of the speeder I need to measure the angular position (not the angular size) of it, which I denote with the variable β. The scene has a panning camera view, which needs to be fixed (the original trailer is available here) [2].

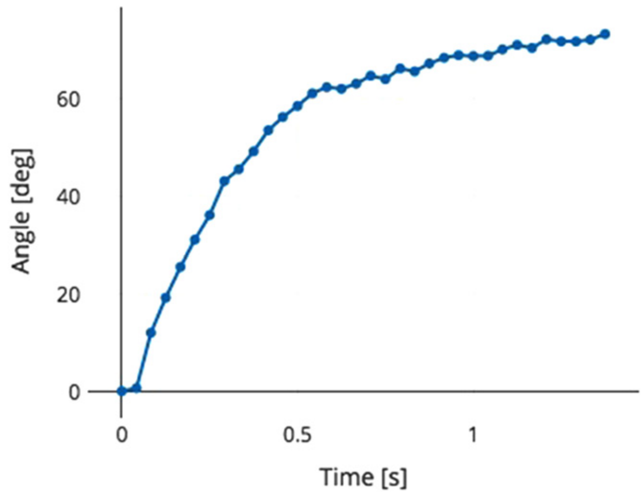

Figure 11.3. Plot of the angular heading of speeder with respect to the camera as a function of time.

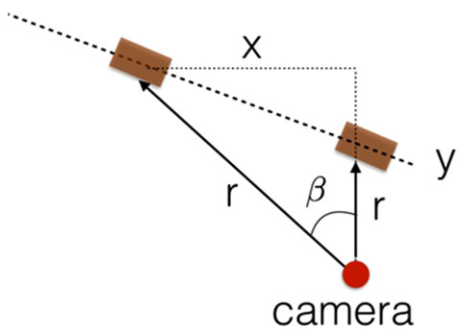

Figure 11.4. Diagram showing the speeder at two different locations along with the camera position.

And now for some data. Since I know both β and r, I can calculate the x and y positions:

$$\begin{aligned} x &= r\sin\beta \\ y &= r\cos\beta. \end{aligned} \tag{11.2}$$

See also figure 11.5.

From this, I can say a few things.

- Both the x and y positions change at a constant rate. This means that the speeder is moving in a straight line with a constant speed.
- Based on the slopes of these two lines, the speeder has an x velocity of 33.3 m s^{-1} and a y velocity of 19.0 m s^{-1}. This gives a total velocity (magnitude) of 38.3 m s^{-1} or 85.7 mph.

- Since both the *x* and *y* velocities seem so linear, I feel fairly comfortable in my calculations.
- There were some guesses involved in this calculation. In particular, I had to guess things like the distance to the camera and the width of the speeder. However, it seems to have worked out reasonably well. If there is an error, it would just be by some factor—but it would still be a speeder moving at a constant velocity.

The data look good, but you do not really understand something until you can make a model of it. Technically, the plots of position versus time are a model, but I would like to make something a little bit better. Here is a numerical model created with GlowScript[1]. The 3D output looks like this figure 11.6).

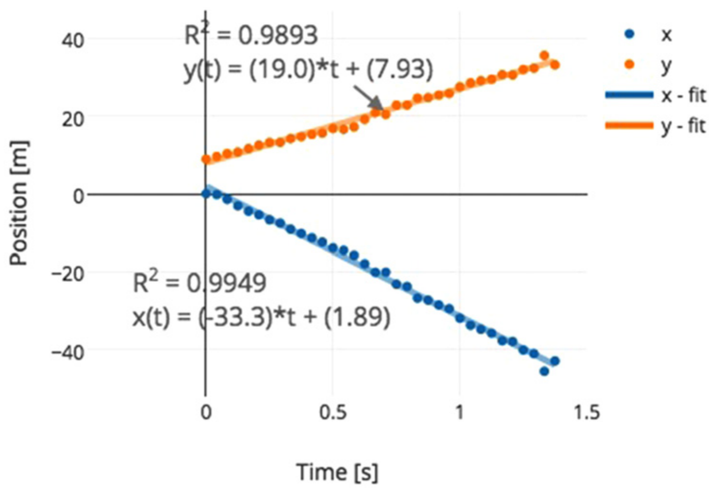

Figure 11.5. x- and y-position of the speeder as a function of time.

Figure 11.6. Screen capture showing the motion of a box representing the speeder as it moves away from the camera.

[1] www.glowscript.org.

It does not look exactly the same as the video. I suspect the difference is due to the field of view of the fake camera in GlowScript.

Now, what about the acceleration? There is one small problem. From the evidence I have gathered so far, the speeder is at rest and then it is moving at a constant speed. It had to accelerate, but the time interval during which it accelerated seems to be very small. Recall the definition of average acceleration in one dimension:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}. \tag{11.3}$$

The speeder starts at rest and then is moving at 38.3 m s^{-1}, so I know the change in velocity. Since the time interval must be very small, the acceleration could be quite high. It all depends on the size of the time interval. If you look at the video, it has a frame rate of 24 frames s^{-1}. In one frame you can see Rey sitting on the speeder. In the next frame, the speeder appears to be moving.

Note that in frame 1 she does not have her goggles on, but she does in frame 2. That could just be a filming error (the goggles should be down in both frames) or perhaps the movie skipped a short time. The shortest this time interval for the acceleration could be is 0.042 s. If you think about how long it takes to pull some goggles down, it could be about 2 s. With a change in velocity of 38.3 m s^{-1}, this gives an acceleration from 911.9 m s^{-2} to 19.15 m s^{-2}. Personally, I am leaning towards the higher value for the acceleration because of the sound effects. If you listen to the trailer you will hear the engine starting in frame 1 and continuing into frame 2. This seems to suggest there was no time cut.

So what if the acceleration is indeed 911 m s^{-2}? This would be a G-force of 92.9 g. That would kill you. Perhaps Rey is a Jedi and can handle this. If the time interval is 2 s, the G-force would be almost 2 g. A normal human would have a tough time holding on to the speeder in that case—but I think it would still be possible.

In this last video analysis example, I have shown a combination of several techniques. First, the primary object (the speeder) moves away from the camera so that we would have to use the angular size to determine the position. However, the speeder does not move directly away from the camera, but instead at a diagonal. By looking at both the angular size and the orientation of the speeder, we can obtain position data for the object.

The second aspect of the analysis deals with the physics of an accelerating human. Overall, it was an excellent and challenging video for analysis.

Now that you have seen several video analysis techniques and examples, it is your turn. Start up your favorite internet browser and find some interesting videos. Or better yet, get your camera and make your own videos. Perform an analysis and then share your findings with others. This is what video analysis is all about.

References

[1] https://en.wikipedia.org/wiki/Daisy_Ridley
[2] http://www.youtube.com/watch?v=erLk59H86ww